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Research Article

A Note on Central Collineations in Fuzzy and Intuitionistic Fuzzy Projective Planes

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Abstract

In this paper, the fuzzy counterparts and the intuitionistic fuzzy counterparts of the central collineations defined in the classical projective planes are introduced in the fuzzy and intuitionistic fuzzy projective planes, respectively. Some basic properties of fuzzy and intuitionistic fuzzy projective planes under the types of central fuzzy and intuitionistic fuzzy collineations are given depending on the base point, the base line and the membership degrees of the fuzzy and intuitionistic fuzzy projective planes.

Keywords: Central collineation, Central fuzzy collineation, Fuzzy projective plane, Central intuitionistic fuzzy collineation, Intuitionistic fuzzy projective plane.

Bulanık ve Sezgisel Bulanık Projektif Düzlemlerde Merkezsel Kolinasyonlar Üzerine Bir Not

Öz

Bu makalede, projektif düzlemlerde tanımlanan merkezsel kolinasyonların bulanık projektif düzlemlerde ve sezgisel bulanık projektif düzlemlerdeki karşılıkları sırasıyla bulanık merkezsel kolinasyonlar ve sezgisel bulanık merkezsel kolinasyonlar olarak sunulmaktadır. Bulanık ve sezgisel bulanık merkezsel kolinasyonların türleri altında, bulanık ve sezgisel bulanık projektif düzlemlerin sağladıkları bazı temel özellikler, bulanık ve sezgisel bulanık projektif düzlemin taban noktası, taban doğrusu ve üyelik derecelerine bağlı olarak verilmektedir.

Anahtar Kelimeler: Merkezsel kolinasyon, Bulanık merkezsel kolinasyon, Bulanık projektif düzlem, Sezgisel bulanık merkezsel kolinasyon, Sezgisel bulanık projektif düzlem.

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1. Introduction

A method of studying projective planes started with a paper by Baer in 1942, which pointed out the close relationship between the Desargues theorem and the existence of central collineations [5]. The fuzzy concept was first proposed by Zadeh in 1965 [15], and many scientists have contributed to this field. The first article on fuzzy groups was published by Azriel Rosenfeld in 1971 [14]. Fuzzy vector spaces were introduced by Katsaras and Liu in 1977 [8]. The fuzzy correspondings of the maps in the vector space was first given by Abdulhalikov in 1996 [1]. In addition, it has been studied by Abdulhalikov that the fuzzy subspace of fuzzy linear maps is isomorphic to the fuzzy subspace of dual maps. Projective planes have been fuzzified by Kuijken et al., see [9]. Also a fuzzy group corresponding to the fuzzy projective geometry was created, so that through these fuzzy projective geometries a relationship between fuzzy vector spaces and fuzzy groups was obtained by Kuijken, Maldeghem and Kerre in 1999 [10]. The fuzzy projective plane collineations were described by Kuijken and Maldeghem in 2003 [12]. As a generalization of Zadeh's Fuzzy Sets, Intuitionistic Fuzzy Set which is characterized by a membership function and a nonmembership function was proposed by Atannassov [4]. In 2009, a new model of intuitionistic fuzzy projective geometry was constructed by Ghassan [6].

The aim of this study is to give the fuzzy and intuitionistic fuzzy counterparts of central collineations defined in the classical projective planes in the fuzzy projective planes and the intuitionistic fuzzy projective planes, respectively and to determine some basic properties under the fuzzy collineation and the intuitionistic fuzzy collineations.

2. Preliminaries

Firstly, some relevant definitions of fuzzy set theory, fuzzy vector space, fuzzy projective space, intuitionistic fuzzy set theory, intuitionistic fuzzy vector space and intuitionistic fuzzy projective space and the collineations are reminded. First recall that fuzzy sets were introduced by Zadeh in the fundamental paper [11].

Definition 2.1. [11] A fuzzy set λ of a set X is a function $\lambda: X \to [0,1]: x \to \lambda(x)$. The number $\lambda(x)$ is called the degree of membership of the point x in λ. The intersection $\lambda \wedge \mu$ of the two fuzzy sets λ and μ on X is given by the fuzzy set $\lambda \wedge \mu: X \to [0,1]: \lambda(x) \wedge \mu(x)$, where $\wedge$ denotes the minimum operator and also $\vee$ denotes the maximum opearator.

Definition 2.2. [4] Let X be a nonempty fixed set. An intuitionistic fuzzy set A on X is an object having the form $A = \{\langle x, \lambda(x), \mu(x) \rangle : x \in X\}$ where the function $\lambda: X \to I$ denote the degree of membership (namely, $\lambda(x)$) and the degree of nonmembership (namely, $\mu(x)$) of each element $x \in X$ to the set A, respectively $0 \leq \lambda(x) + \mu(x) \leq 1$ for each $x \in X$. An intuitionistic fuzzy set A $=\{\langle x, \lambda(x), \mu(x) \rangle : x \in X\}$ can be written in A $=\{\langle x, \lambda, \mu\rangle : x \in X\}$, or simply A $=\langle \lambda, \mu\rangle$.

Let $A = \{\langle x, \lambda(x), \mu(x) \rangle : x \in X\}$ and $B = \{\langle x, \delta(x), \gamma(x) \rangle : x \in X\}$ be an intuitionistic fuzzy sets on X. Then,

(a) $\bar{A} = \{\langle x, \mu(x), \lambda(x) \rangle : x \in X\}$ (the complement of A).

(b) $A \cap B = \{\langle x, \lambda(x) \wedge \delta(x), \mu(x) \vee \gamma(x) \rangle : x \in X\}$ (the meet of A and B).

(c) $A \cup B = \{\langle x, \lambda(x) \vee \delta(x), \mu(x) \wedge \gamma(x) \rangle : x \in X\}$ (the join of A and B).

(d) $A \subseteq B \iff \lambda(x) \leq \delta(x)$ and $,\mu(x) \geq \gamma(x)$ for each $x \in X$.

(e) $A = B \iff A \subseteq B$ and $B \subseteq A$.

(f) $\tilde{1} = \{\langle x, 1, 0 \rangle : x \in X\}$, $\tilde{0} = \{\langle x, 0, 1 \rangle : x \in X\}$.

Definition 2.3. [7] Let $\mu: V \to [0,1]$ be a fuzzy set on V. Then we call μ a fuzzy vector space on V if and only if $\mu(a.\bar{u} + b.\bar{v}) \geq \mu(\bar{u}) \wedge \mu(\bar{v}), \forall \bar{u}, \bar{v} \in V$ and $a, b \in K$.

Definition 2.4. [13] Let $A = (\lambda_A, \mu_A)$ be an intuitionistic fuzzy set of a classical vector space V over F. For any $x, y \in V$ and $\alpha, \beta \in F$, if it satisfy $\lambda_A(\alpha x + \beta y) \geq min\{\lambda_A(x), \lambda_A(y)\}$ and $\mu_A(\alpha x + \beta y) \leq max\{\mu_A(x), \mu_A(y)\}$, then A is called an intuitionistic fuzzy subspace of V. Let V_n denote the set of all n-tuples $(\langle x_{1\lambda}, x_{1\mu}\rangle), \langle x_{2\lambda}, x_{2\mu}\rangle, \cdots, \langle x_{n\lambda}, x_{n\mu}\rangle$ over F. An element of V_n is called an intuitionistic fuzzy vector (IFV) of dimension n, where $x_{i\lambda}$ and $x_{i\mu}$ are the membership and non-membership values of the component x_i.

Definition 2.5. [7] Suppose $\mathcal{P}$ is an n-dimensional projective space. A fuzzy set λ on the point set of $\mathcal{P}$ is a fuzzy n-dimensional projective space on $\mathcal{P}$ if $\lambda(p) \geq \lambda(r)$, for all collinear points p, q, r of $\mathcal{P}$. We denotes as $(\lambda, \mathcal{P})$. The projective space $\mathcal{P}$ is called the underlying (crisp) projective space of $(\lambda, \mathcal{P})$. If $\mathcal{P}$ is a fuzzy point, line, plane, etc., we use underlying point, underlying line, underlying plane, etc., respectively. We will sometimes briefly write λ instead of $(\lambda, \mathcal{P})$.

Definition 2.6. [6] An intuitionistic fuzzy set $A = \{\langle x, \lambda(x), \mu(x) \rangle : x \in X\}$ on n-dimensional projective space S is an intuitionistic fuzzy n-dimensional projective space on S if $\lambda(p) \geq \lambda(q) \wedge \lambda(r)$ and $\mu(p) \geq \mu(q) \vee \mu(r)$, for any three collinear points p, q, r of A we denoted $[A, S]$. The projective space S is called the base projective space of $[A, S]$ if $[A, S]$ is an intuitionistic fuzzy point, line, plane, … , we use base point, base line, base plane, … , respectively.

In practice, this means in the point set of a line, all elements have the same degree of membership, but may not be the same. Moreover, more generally speaking, this means that in any subspace U, all points have the same degree of membership, except that they may be in subspace U′ of U. All points have the same degree of membership, except for those that may be in a subspace U″ of U′ , etc. [2].

Definition 2.7. [2] Let $(\lambda, \mathcal{P})$ be a fuzzy projective space and let U be a subspace of $\mathcal{P}$. Then (λ_U, U) is called a fuzzy subspace of $(\lambda, \mathcal{P})$ if $\lambda_U(x) \leq \lambda(x)$ for $x \in U$, and $\lambda_U(x) = 0$ for $x \notin U$.

Definition 2.8. [3] Let $\langle \lambda, \mu\rangle$ be an intuitionistic fuzzy projective space and let U be a subspace of $\mathcal{P}$. Then (λ_U, μ_U, U) is called a intuitionistic fuzzy subspace of $(\lambda, \mathcal{P})$ if $\lambda_U(x) \leq \lambda(x)$ and $\mu_U(x) \geq \mu(x)$ for $x \in U$, and $\lambda_U(x) = 0, \mu_U(x) = 1$ for $x \notin U$.

Definition 2.9. [9] Let $(\lambda, \mathcal{P})$ be a fuzzy projective space of dimension n. Then there are constants $a_i \in [0,1], i = 0,1, \dots, n$, with $a_i \geq a_{i+1}$, and a chain of subspaces $(U_i)_{0 \leq i \leq n}$ with $U_i \subseteq U_{i+1}$ and $dim U_i = i$, such that

$$\begin{aligned} \lambda: \quad & \mathcal{P} \to [0,1] \\ & x \to a_0 \qquad for\, x \in U_0, \\ & x \to a_i \qquad for\, x \in U_i \backslash U_{i-1}, \quad i = 1,2, \dots, n. \end{aligned}$$

Definition 2.10. [3] Let (λ, μ) be an intuitionistic fuzzy projective space of dimension n. Then there are constants $a_i, b_i \in [0,1], i = 0,1,\dots,n$, with $a_i + b_i \leq 1$, and a chain of subspaces $(U_i)_{0\leq i\leq n}$ with $U_i \sqsubseteq U_{i+1}$ and $dimU_i = i$, such that

$$\begin{aligned}(\lambda,\mu):\ & \mathcal{P} \rightarrow [0,1]\times[0,1]\\ & \bar{u} \rightarrow (a_0, b_0), \quad for\ \bar{u} \in U_0,\\ & \bar{u} \rightarrow (a_i, b_i), \quad for\ \bar{u} \in U_i \backslash U_{i-1}, \quad i = 1,2,\dots,n.\end{aligned}$$

Definition 2.11. [9] Consider the projective plane $\mathcal{P} = (N, D, \circ)$.

Suppose $p \in N$ and $\alpha \in [0,1]$. The fuzzy point (p, α) is the following fuzzy set on the point set N of $\mathcal{P}$:

$$\begin{aligned}(p,\alpha):\ & N \rightarrow [0,1]\\ & p \rightarrow \alpha\\ & x \rightarrow 0 \quad if\ x \in N\backslash\{p\}.\end{aligned}$$

The point p is called the base point of the fuzzy point (p, α).

A fuzzy line (L, β) with base line L is defined in a similar way.

Two fuzzy lines (L, α) and (M, β), with $\alpha \wedge \beta > 0$, intersect in the unique fuzzy point $(L \cap M, \alpha \wedge \beta)$. Dually, the fuzzy points (p, λ) and (q, μ) with $\lambda \wedge \mu > 0$, span the unique fuzzy line $(\langle p, q\rangle,\ \lambda \wedge \mu)$.

Definition 2.12. [6] Consider the projective plane $\mathcal{P} = (N, D, \circ)$. Suppose $\alpha \in N$ and $\alpha, \beta \in [0,1]$. The IF-point (a, α, β) is the following intuitionistic fuzzy set on the point set N of $\mathcal{P}$:

$$(a,\alpha,\beta): N \rightarrow [0,1] \times [0,1]: \begin{cases} a \rightarrow \alpha, a \rightarrow \beta \\ x \rightarrow 0, \quad x \in N\backslash\{a\} \end{cases}$$

The point $\alpha \in N$ is called the base point of the IF-point (a, α, β). An IF-line (L, α, β) with base line L is defined in a similar way.

The IF-lines (L, α, β) and (M, σ, ω) intersect in the unique IF-point $(L \cap M, \alpha \wedge \sigma, \beta \vee \omega)$.

Definition 2.13. [11] Suppose $\mathcal{P}$ is a projective plane $\mathcal{P} = (N, D, \circ)$. The fuzzy set $(\lambda, \mathcal{P})$ on $N \cup D$ is a fuzzy projective plane on $\mathcal{P}$ if

i) $\lambda(L) \geq \lambda(p) \wedge \lambda(q), \forall\, p, q: \langle p, q\rangle = L$ and
ii) $\lambda(p) \geq \lambda(L) \wedge \lambda(M), \forall\, L, M: L \cap M = p.$

Definition 2.14. [6] Suppose $\mathcal{P}$ is a projective plane $\mathcal{P} = (N, D, \circ)$. The intuitionistic fuzzy set $Z = \langle \lambda, \mu\rangle$ on $N \cup D$ is an intuitionistic fuzzy projective plane on $\mathcal{P}$ if :

i) $\lambda(L) \geq \lambda(p) \wedge \lambda(q)$ and $,\mu(L) \leq \mu(p) \vee \mu(q)$; $\forall\, p, q: \langle p, q\rangle = L$ and
ii) $\lambda(p) \geq \lambda(L) \wedge \lambda(M)$ and $\mu(p) \leq \mu(L) \vee \mu(M)$; $\forall\, L, M: L \cap M = p$.

The intuitionistic fuzzy projective plane can be considered as an ordinary projective plane, where to every point (and only to points) one (and only one) degrees of membership and non-membership are assigned.

Definition 2.15. [1] Let E and L be vector spaces over the same field F, and let $\mu: E \rightarrow [0,1]$, $\lambda: L \rightarrow [0,1]$ be fuzzy subspaces. If $\lambda(\varphi(x)) \geq \mu(x)$ for all $x \in E$, we say that a linear map $\varphi: E \rightarrow L$ is fuzzy linear from the fuzzy subspace μ to fuzzy subspace λ. The space of fuzzy linear maps from μ to λ is denoted by $FHom(\mu, \lambda)$.

Definition 2.16. [1] Let (E_1, μ_1) and (E_2, μ_2) be two fuzzy vector spaces. If there exists an isomorphism $\varphi: E_1 \rightarrow E_2$ with the property $\mu_1(x) = \mu_2(\varphi(x))$ for all $, x \in E_1, \mu_1: E_1 \rightarrow [0,1]$ and $\mu_2: E_2 \rightarrow [0,1]$ are isomorphic.

Now here, the intuitionistic fuzzy counterparts of the theorems and the proofs related to the fuzzy linear maps in Abdulhalikov's works [1] are given by using the intuitionistic fuzzy linear maps definition.

Definition 2.17. [3] Let V and W be two vector spaces over the same field F and T be a linear map from V to W. Suppose that (V, λ_V, μ_V) and (W, λ_W, μ_W) are intuitionistic fuzzy vector spaces on F. For all, $x \in V$, if

$\lambda_W(T(x)) \geq \lambda_V(x)$ and $\mu_W(T(x)) \leq \mu_V(x)$

is satisfied such that $0 \leq \lambda_V + \mu_V \leq 1$ and $0 \leq \lambda_W + \mu_W \leq 1, T$ is called as an intuitionistic fuzzy linear maps from the intuitionistic fuzzy vector space (V, λ_V, μ_V) to the intuitionistic fuzzy vector space (W, λ_W, μ_W).

Definition 2.18. [3] Let $[\mathcal{P}, \lambda]$ and $[\mathcal{P}', \mu]$ be two fuzzy projective planes with base planes $\mathcal{P} = (N, D, \circ)$, $P' = (N', D', \circ')$ respectively. Suppose that f is a homomorphism from $\mathcal{P}$ to $\mathcal{P}'$. $\bar{f}$ is called as the fuzzy homomorphism from $[\mathcal{P}, \lambda]$ to $[\mathcal{P}', \mu]$, if $\bar{f}(p, \alpha) = (f(p), \beta)$ for all the points $(p, \alpha) \in [\mathcal{P}, \lambda]$ where $\lambda(p) = \alpha$, $\mu(f(p)) = \beta$ and $\alpha \leq \beta$. If f is an isomorphism of $\mathcal{P}$ into $\mathcal{P}'$ and $\alpha = \beta$, then $\bar{f}$ is called as the fuzzy isomorphism between the fuzzy projective planes $[\mathcal{P}, \lambda]$ and $[\mathcal{P}', \mu]$. Also if $\mathcal{P} = \mathcal{P}'$, the fuzzy isomorphism $\bar{f}$ is called as the fuzzy collineation of $[\mathcal{P}, \lambda]$.

Definition 2.19. [3] Let $[\mathcal{P}, \lambda_{\mathcal{P}}, \mu_{\mathcal{P}}]$ and $[\mathcal{P}', \lambda_{\mathcal{P}'}, \mu_{\mathcal{P}'}]$ be two intuitionistic fuzzy projective planes with base planes $\mathcal{P} = (N, D, \circ)$, $P' = (N', D', \circ')$ respectively. Suppose that f is a homomorphism from $\mathcal{P}$ to $\mathcal{P}'$. $\bar{f}$ is called as the intuitionistic fuzzy homomorphism from $[\mathcal{P}, \lambda_{\mathcal{P}}, \mu_{\mathcal{P}}]$ to $[\mathcal{P}', \lambda_{\mathcal{P}'}, \mu_{\mathcal{P}'}]$, if $\bar{f}(p, \alpha, \beta) = (f(p), \alpha', \beta')$ for all the points $(p, \alpha, \beta) \in [\mathcal{P}, \lambda_{\mathcal{P}}, \mu_{\mathcal{P}}]$ where $\lambda_{\mathcal{P}}(p) = \alpha, \mu_{\mathcal{P}}(f(p)) = \beta$, $\lambda_{\mathcal{P}'}(p) = \alpha'$, $\mu_{\mathcal{P}'}(f(p)) = \beta'$ and $\alpha \leq \alpha'$, $\beta \geq \beta'$. If f is an isomorphism of $\mathcal{P}$ to $\mathcal{P}'$ and $\alpha = \alpha'$, $\beta = \beta'$, then $\bar{f}$ is called as the intuitionistic fuzzy isomorphism between the fuzzy projective planes $[\mathcal{P}, \lambda_{\mathcal{P}}, \mu_{\mathcal{P}}]$ and $[\mathcal{P}', \lambda_{\mathcal{P}'}, \mu_{\mathcal{P}'}]$. Also if $\mathcal{P} = \mathcal{P}'$, the intuitionistic fuzzy isomorphism $\bar{f}$ is called as the intuitionistic fuzzy collineation of $[\mathcal{P}, \lambda_{\mathcal{P}}, \mu_{\mathcal{P}}]$.

3. Central Fuzzy Collineations in Fuzzy Projective Planes

The fuzzy counterparts of the central collineations of the projective planes are given in [12]. In this section, we investigate some basic properties of fuzzy projective planes under the types of central fuzzy collineations are given depending on the base point, the base line and the membership degrees of the fuzzy projective planes.

Definition 3.1. Let $[\boldsymbol{\mathcal{P}}, \boldsymbol{\lambda}]$ be fuzzy projective plane with base plane $\boldsymbol{\mathcal{P}} = (\boldsymbol{N}, \boldsymbol{D}, \circ)$ and $\bar{\boldsymbol{f}}$ determined by the collineation $\boldsymbol{f}$ in $\boldsymbol{\mathcal{P}}$ be the fuzzy collineation of $[\boldsymbol{\mathcal{P}}, \boldsymbol{\lambda}]$. The fuzzy point $(\boldsymbol{p}, \boldsymbol{\alpha})$ is called as the center of $\bar{\boldsymbol{f}}$ if every fuzzy line passing through $(\boldsymbol{p}, \boldsymbol{\alpha})$ remains invariant under the fuzzy collineation $\bar{\boldsymbol{f}}$ in $[\boldsymbol{\mathcal{P}}, \boldsymbol{\lambda}]$. The fuzzy line $(\boldsymbol{E}, \boldsymbol{\beta})$ is called as an axis of the fuzzy collineation, if every fuzzy point on the fuzzy line $(\boldsymbol{E},\boldsymbol{\beta})$ is invariant under the fuzzy collineation $\bar{\boldsymbol{f}}$.

The fuzzy collineation $\bar{\boldsymbol{f}}$ which has the center point $(\boldsymbol{m},\boldsymbol{\alpha})$ and the axis $(\boldsymbol{E},\boldsymbol{\beta})$ is called a $\left((\boldsymbol{m},\boldsymbol{\alpha}),(\boldsymbol{E},\boldsymbol{\beta})\right)$- central fuzzy collineation. If the center point is on the axis, $\bar{\boldsymbol{f}}$ is called as a fuzzy elation and if the center point is not on the axis, then $\bar{\boldsymbol{f}}$ is called as a fuzzy homology.

For the fuzzy unit collineation, each fuzzy point as a center and each fuzzy line as an axis can be considered in $[\boldsymbol{\mathcal{P}},\boldsymbol{\lambda}]$. Therefore, the fuzzy unit collineation is both a fuzzy elation and a fuzzy homology.

Since the fuzzy lines passing through the center are invariant under the fuzzy collineation $\bar{\boldsymbol{f}}$, the center, a point and its image are fuzzy collinear. But the fuzzy points on the fuzzy lines passing through the center can replaced by each other.

Theorem 3.2. Each fuzzy homology of Fuzzy Fano plane is an unit collineation.

Proof. Let $\bar{\boldsymbol{f}}$ be a fuzzy homology of Fuzzy Fano plane $[\boldsymbol{\mathcal{P}},\boldsymbol{\lambda}]$ with $(\boldsymbol{m},\boldsymbol{\alpha})$-center and $(\boldsymbol{E},\boldsymbol{\beta})$-axis. Since $\bar{\boldsymbol{f}}$ is a fuzzy homology, the center point $\boldsymbol{m}$ is not on the axis in the base projective plane $\boldsymbol{\mathcal{P}}$ defined by collineation $\boldsymbol{f}$. Suppose that $\bar{\boldsymbol{f}}$ is a fuzzy collineation different from the fuzzy unit collineation. The center $(\boldsymbol{m},\boldsymbol{\alpha})$ and each point on the axis $(\boldsymbol{E},\boldsymbol{\beta})$ are invariant under $\bar{\boldsymbol{f}}$. All points of Fuzzy Fano plane are on the fuzzy lines intersect with the axis. So the center and the intersection point on the axis are invariant under $\bar{\boldsymbol{f}}$. There remains a fuzzy point on the fuzzy line passing through $(\boldsymbol{m},\boldsymbol{\alpha})$ such that its image is undefined. Since the center, the point and its image are fuzzy collinear, it has to turn into itself under $\bar{\boldsymbol{f}}$. In this case, each point remains invariant. So $\bar{\boldsymbol{f}}$ is fuzzy unit collineation.

Theorem 3.3. Fuzzy Fano plane has only one central fuzzy collineation different from the unit collineation.

Proof. Let $\bar{f}$ be a central fuzzy collineation of Fuzzy Fano plane different from the unit collineation. From previous theorem, if $\bar{f}$ is a fuzzy homology, then $\bar{f}$ is a fuzzy unit collineation. Suppose that $\bar{f}$ is a fuzzy elation. Then the center is on the axis of $\bar{f}$. The image of any fuzzy point not on the axis must be on a fuzzy line joining it to the center. Since $\bar{f}$ is different from fuzzy unit collineation, there are two remaining fuzzy points on the fuzzy line which must match each other under $\bar{f}$. So there is only one central fuzzy collineation different from unit collineation in Fuzzy Fano projective plane.

Corallary Fuzzy Fano plane has only one fuzzy elation different from the unit collineation.

From now on, we considered the fuzzy projective plane $[\mathcal{P},\lambda]$ with base plane $\mathcal{P}$ and λ in the following form:

$$\begin{aligned} \lambda:\ \ \mathcal{P} &\rightarrow [0,1] \\ q &\rightarrow a_0, \\ p &\rightarrow a_1, \quad p \in L\backslash\{q\} \\ p &\rightarrow a_2, \quad p \in \mathcal{P}\backslash\{L\} \end{aligned}$$

where $\boldsymbol{L}$ is a projective line of $\boldsymbol{\mathcal{P}}$ contains $\boldsymbol{q}$ and $\boldsymbol{a_0} \geq \boldsymbol{a_1} \geq \boldsymbol{a_2}$, $\boldsymbol{a_i} \in [\boldsymbol{0},\boldsymbol{1}]$, i=0, 1, 2.

The fuzzy point $(\boldsymbol{q},\boldsymbol{a_0})$ and the fuzzy line $(\boldsymbol{L},\boldsymbol{a_1})$ are called as the base point, the base line of the fuzzy projective plane [P, λ], respectively. Some properties under the central fuzzy collineation in $[\boldsymbol{\mathcal{P}},\boldsymbol{\lambda}]$ depending on the base line, the base point and the membership degrees of $[\boldsymbol{\mathcal{P}},\boldsymbol{\lambda}]$ are introduced with the following theorems.

Theorem 3.4. Suppose that $\bar{\boldsymbol{f}}$ is a central fuzzy collineation of $[\boldsymbol{\mathcal{P}},\boldsymbol{\lambda}]$ defined by the collineation *f* of the base plane $\boldsymbol{\mathcal{P}}$ of order *n* with $\boldsymbol{m}$*-center* and *E-axis*.

i) The center $(\boldsymbol{m},\boldsymbol{\alpha})$ is invariant under the central fuzzy collineation $\bar{\boldsymbol{f}}$.

ii) The axis $(\boldsymbol{E},\boldsymbol{\beta})$ is invariant under the central fuzzy collineation $\bar{\boldsymbol{f}}$.

Proof. i) Let $(\boldsymbol{m},\boldsymbol{\alpha})$ be center of $\bar{\boldsymbol{f}}$ in $[\boldsymbol{\mathcal{P}},\boldsymbol{\lambda}]$. The lines $\boldsymbol{L_i}, \boldsymbol{i}=\boldsymbol{1},\boldsymbol{2},\dots,\boldsymbol{n}+\boldsymbol{1}$ passing through the center are invariant from the definition of the center of f in $\boldsymbol{\mathcal{P}}$. Let the center $(\boldsymbol{m},\boldsymbol{\alpha})$ be the intersection point of the fuzzy lines $(\boldsymbol{L_i},\boldsymbol{\beta_i}), \boldsymbol{i}=\boldsymbol{1},\boldsymbol{2},\dots,\boldsymbol{n}+\boldsymbol{1}$ in $[\boldsymbol{\mathcal{P}},\boldsymbol{\lambda}]$.

$$(\boldsymbol{m},\boldsymbol{\alpha}) = \bigcap_{i=1}^{n+1}(\boldsymbol{L_i},\boldsymbol{\beta_i}) = \left(\bigcap_{i=1}^{n+1}\boldsymbol{L_i}\ ,\bigwedge_{i=1}^{n+1}\boldsymbol{\beta_i}\ \right)$$

By using definition $\bar{\boldsymbol{f}}$,

$$\begin{aligned} \bar{\boldsymbol{f}}\,(\boldsymbol{m},\boldsymbol{\alpha}) = (\boldsymbol{f}(\boldsymbol{m}),\boldsymbol{\alpha}) &= \boldsymbol{f}\left(\bigcap_{i=1}^{n+1}\boldsymbol{L_i}\ ,\bigwedge_{i=1}^{n+1}\boldsymbol{\beta_i}\ \right) \\ &= \left(\boldsymbol{f}\left(\bigcap_{i=1}^{n+1}\boldsymbol{L_i}\right),\bigwedge_{i=1}^{n+1}\boldsymbol{\beta_i}\ \right) = \left(\bigcap_{i=1}^{n+1}\boldsymbol{f}(\boldsymbol{L_i}),\ \boldsymbol{\alpha}\right) \\ &= \left(\bigcap_{i=1}^{n+1}\boldsymbol{L_i}\,,\boldsymbol{\alpha}\ \right) = (\boldsymbol{m},\boldsymbol{\alpha}) \end{aligned}$$

is obtained. So the center $(\boldsymbol{m},\boldsymbol{\alpha})$ is invariant under $\bar{\boldsymbol{f}}$.

ii) For all $(\boldsymbol{p_i},\boldsymbol{\alpha_i}), \boldsymbol{i}=\boldsymbol{1},\boldsymbol{2},\dots,\boldsymbol{n}+\boldsymbol{1}$ on the axis, from the definition of the axis of the central fuzzy collineation $\bar{\boldsymbol{f}}$,

$$\bar{\boldsymbol{f}}\,(\boldsymbol{p_i},\boldsymbol{\alpha_i}) = (\boldsymbol{p_i},\boldsymbol{\alpha_i}), \qquad \boldsymbol{i}=\boldsymbol{1},\boldsymbol{2},\dots,\boldsymbol{n}+\boldsymbol{1}.$$

The axis $\boldsymbol{E}$ can written $\boldsymbol{E}=\cup_{i=1}^{n+1}\boldsymbol{p_i}$ in $\boldsymbol{\mathcal{P}}$ and

$(\boldsymbol{E},\boldsymbol{\beta}) = \langle\cup_{i=1}^{n+1}\boldsymbol{p_i}\ ,\wedge_{i=1}^{n+1}\boldsymbol{\alpha_i}\ \rangle$ in $[\boldsymbol{\mathcal{P}},\boldsymbol{\lambda}]$. By using the definition of the fuzzy collineation $\bar{\boldsymbol{f}}$ in $[\boldsymbol{\mathcal{P}},\boldsymbol{\lambda}]$,

$$\bar{f}(E,\beta)=\bar{f}\left(\langle\bigcup_{i=1}^{n+1}p_i,\bigwedge_{i=1}^{n+1}\alpha_i\rangle\right)=\langle f\left(\bigcup_{i=1}^{n+1}p_i\right),\bigwedge_{i=1}^{n+1}\alpha_i\rangle$$
$$=\langle\bigcup_{i=1}^{n+1}f(p_i),\bigwedge_{i=1}^{n+1}\alpha_i\rangle=\langle\bigcup_{i=1}^{n+1}p_i,\bigwedge_{i=1}^{n+1}\alpha_i\rangle$$
$$=\left(E,\bigwedge_{i=1}^{n+1}\alpha_i\right)=(E,\beta)$$

is obtained. So the axis (E,β) is invariant under $\bar{f}$.

Theorem 3.5. Suppose that $\bar{f}$ is a central fuzzy collineation with (m,α) −center of $[\mathcal{P},\lambda]$. If $\bar{f}$ leaves two distinct lines that do not pass through the center invariant, then $\bar{f}$ is the fuzzy unit collineation.

Proof. Let (L_1,β_1) and (L_2,β_2) be two fuzzy invariant line that do not pass through the center of $\bar{f}$ in $[\mathcal{P},\lambda]$. From [3], the intersection point of these lines remains invariant under $\bar{f}$. Since the center and every fuzzy line passing through the center are invariant under $\bar{f}$, the intersection points of the fuzzy lines (L_1,β_1) and (L_2,β_2) with the lines passing through the center (m,α) are invariant under $\bar{f}$. So, every point on $(L_i,\beta_i), i=1,2$ is invariant and it is well shown that if two fuzzy points that are not on a fuzzy line which is pointwise are invariant, $\bar{f}$ is the unit in [3]. So the proof is completed.

Theorem 3.6. Suppose that $\bar{f}$ is a central fuzzy collineation of $[\mathcal{P},\lambda]$, defined by the collineation f of the base plane $\mathcal{P}$. Then,

i) Let f be a homology such that its center m is the base point on the base line L. If **the fuzzy collineation** $\bar{f}$ with the center (q,a_0) has an axis, then $a_1=a_2$.

ii) Let f be a homology such that its center m is on the base line L different from the base point. If **the central fuzzy collineation** $\bar{f}$ with the center (m,a_1) has an axis, then there are the relationships $a_1=a_2$ or $a_0=a_1=a_2$ among the membership degrees of $[\mathcal{P},\lambda]$.

iii) Let f be a homology such that its center m not on the base line L. If the fuzzy collineation $\bar{f}$ has an axis (E,a_2), then there are relationships $a_1=a_2$ or $a_0=a_1=a_2$ in $[\mathcal{P},\lambda]$.

Proof. i) Let f be (q,E)-homology.

Since the center q is not on the axis, $L\neq E$.

Let **the fuzzy collineation** $\bar{f}$ with the center (q,a_0) have an axis. Suppose that the axis E has a_2 membership degree different from the fuzzy base line (L,a_1). Let's take (p_1,a_1) and (p_2,a_2) be on the axis (E,a_2) such that $p_1=qp_1\cap E$.

$$\bar{f}(p_1,a_1)=\bar{f}\left((qp_1,a_1)\cap(E,a_2)\right)=(qp_1\cap E,a_2)=(p_1,a_2).$$

Since (E,a_2) is the axis of the fuzzy collineation $\bar{f}$, (p_1,a_1) must be invariant. In this case, $a_1=a_2$ is obtained.

ii) Let f be (m,E)-homology such that $m\circ L$, $m\neq q$.

Case 1: Let the axis E intersect the base line on a point different from the base point q.

Suppose that the axis of $\bar{f}$ is (E,a_2).

Let's take the fuzzy points (p_1,a_1) and (p_2,a_2) be on the axis (E,a_2) such that $p_1=mp_1\cap E$. Then, $\bar{f}(p_1,a_1)$ is $\bar{f}\left((mp_1,a_1)\cap(E,a_2)\right)$=$(f(mp_1\cap E),a_1\wedge a_2)$=$(p_1,a_2)$. Since (p_1,a_1) is on the axis, it is invariant. Hence, $a_1=a_2$ is obtained.

Case 2: Let the axis E of f intersect the base line $\mathbf{L}$ on the base point q.

Let's take the fuzzy points (q,a_0) and (p,a_2) be on the axis (E,a_2) such that $q=mq\cap E$. Then, $\bar{f}(q,a_0)$ is $\bar{f}\left((mq,a_1)\cap(E,a_2)\right)$=$(f(mq\cap E),a_1\wedge a_2)$=$(mq\cap E,a_2)=(q,a_2)$. Since (q,a_0) is on the axis, so it is fuzzy invariant. $a_0=a_2$ is obtained.

From the relationship $a_0\geq a_1\geq a_2$ among the membership degrees in $[\mathcal{P},\lambda]$, $a_0=a_1=a_2$ is obtained.

iii) Suppose that f is (m,E)- homology, $m\not\circ L$. Two results are obtained. The axis can be the base line (L,a_1) or different from the base line.

If the axis (E,a_2) of $\bar{f}$ is the base line (L,a_1), it is easily seen that $a_0=a_1=a_2$.

When the axis (E,a_2) of $\bar{f}$ is different from the base line (L,a_1), it intersects the base line in the base point (q,a_0) or a point (p,a_1) on the base line (L,a_1). If the intersection point is the base point (q,a_0), the relationship $a_0=a_1=a_2$ is obtained easily. Similarly, if the intersection point is not the base point (q,a_0) on the base line, the equation $a_1=a_2$ among the membership degrees is obtained.

Corallary Suppose that $\bar{f}$ is a central fuzzy collineation of $[\mathcal{P},\lambda]$ defined by the $(m,E)-\textit{homology}\ f$ of the base plane $\mathcal{P}$ such that the base point q is on the axis E, then there is only one membership degree in $[\mathcal{P},\lambda]$.

Theorem 3.7. Suppose that the collineation f of the base plane $\mathcal{P}$ is a (m,E)−central collineation. Then, the fuzzy collineation $\bar{f}$ of $[\mathcal{P},\lambda]$ defined by the collineation of f is central fuzzy collineation, too.

Proof. Let f be (m,E)−central collineation of the base plane $\mathcal{P}$. Since E is the axis of f, E is pointwise. From the definition of the fuzzy collineation $\bar{f}$, the fuzzy line (E,β) is pointwise. So the fuzzy line (E,β) is the axis for the fuzzy collineation $\bar{f}$. Similarly, since m is the axis of f, m is linewise. Using the definition $\bar{f}$, $\bar{f}(m,\alpha)=(f(m),\alpha)=(m,\alpha)$ and for any fuzzy line $(L_i,\beta_i), i=1,2,\ldots,n+1$ passing through (m,α), $\bar{f}(L_i,\beta_i)=(f(L_i),\beta_i)=(L_i,\beta_i)$. Hence the fuzzy point (m,α) is the center for the fuzzy collineation $\bar{f}$.

Consequently, the fuzzy collineation $\bar{f}$ is a $\left((m,\alpha),(E,\beta)\right)-$ central fuzzy collineation.

Corollary There is a central fuzzy collineation determined by the central collineation $\boldsymbol{f}$ of the base plane $\boldsymbol{\mathcal{P}}$ in the fuzzy projective plane $[\boldsymbol{\mathcal{P}}, \boldsymbol{\lambda}]$.

4. Central Intuitionistic Fuzzy Collineations in Intuitionistic Fuzzy Projective Planes

In this section, the definition of the central fuzzy collineations in the fuzzy projective planes is extended to the central intuitionistic fuzzy collineations in the intuitionistic fuzzy projective planes and some basic properties of these are introduced.

Definition 4.1. Let $[\boldsymbol{\mathcal{P}}, \boldsymbol{\lambda}, \boldsymbol{\mu}]$ be the intuitionistic fuzzy projective plane with the base plane $\boldsymbol{\mathcal{P}} = (\boldsymbol{N}, \boldsymbol{D}, \circ)$ and $\overline{\boldsymbol{f}}$ determined by the collineation $\boldsymbol{f}$ in $\boldsymbol{\mathcal{P}}$ be the intuitionistic fuzzy collineation of $[\boldsymbol{\mathcal{P}}, \boldsymbol{\lambda}, \boldsymbol{\mu}]$. The intuitionistic fuzzy point $(\boldsymbol{p}, \boldsymbol{\alpha}, \boldsymbol{\beta})$ is called as the center of $\overline{\boldsymbol{f}}$ if every intuitionistic fuzzy line passing through $(\boldsymbol{p}, \boldsymbol{\alpha}, \boldsymbol{\beta})$ remains invariant under the intuitionistic fuzzy collineation $\overline{\boldsymbol{f}}$ in $[\boldsymbol{\mathcal{P}}, \boldsymbol{\lambda}, \boldsymbol{\mu}]$. The intuitionistic fuzzy line $(\boldsymbol{E}, \boldsymbol{\alpha}', \boldsymbol{\beta}')$ is called as an axis of the intuitionistic fuzzy collineation if every intuitionistic fuzzy point on the intuitionistic fuzzy line $(\boldsymbol{E}, \boldsymbol{\alpha}', \boldsymbol{\beta}')$ is invariant under the intuitionistic fuzzy collineation $\overline{\boldsymbol{f}}$.

The intuitionistic fuzzy collineation $\overline{\boldsymbol{f}}$ which has the center point $(\boldsymbol{m}, \boldsymbol{\alpha}, \boldsymbol{\beta})$ and the axis $(\boldsymbol{E}, \boldsymbol{\alpha}', \boldsymbol{\beta}')$ is called a $\big((\boldsymbol{m}, \boldsymbol{\alpha}, \boldsymbol{\beta}), (\boldsymbol{E}, \boldsymbol{\alpha}', \boldsymbol{\beta}')\big) -$ central intuitionistic fuzzy collineation. If the center point is on the axis, $\overline{\boldsymbol{f}}$ is called as an intuitionistic fuzzy elation and if the center point is not on the axis, then $\overline{\boldsymbol{f}}$ is called as an intuitionistic fuzzy homology.

For the intuitionistic fuzzy unit collineation, each intuitionistic fuzzy point as a center and each intuitionistic fuzzy line as an axis can be considered in $[\boldsymbol{\mathcal{P}}, \boldsymbol{\lambda}, \boldsymbol{\mu}]$. Therefore, the intuitionistic fuzzy unit collineation is both an intuitionistic fuzzy elation and an intuitionistic fuzzy homology.

Since the intuitionistic fuzzy lines passing through the center are invariant under the intuitionistic fuzzy collineation $\overline{\boldsymbol{f}}$, the center, a point and its image are intuitionistic fuzzy collinear. But the intuitionistic fuzzy points on the intuitionistic fuzzy lines passing through the center can replaced by each other under the intuitionistic fuzzy collineation $\overline{\boldsymbol{f}}$.

Theorem 4.2. Each intuitionistic fuzzy homology of Intuitionistic Fuzzy Fano plane is an unit collineation.

Proof. Let $\overline{\boldsymbol{f}}$ be an intuitionistic homology of Intuitionistic Fuzzy Fano plane $[\boldsymbol{\mathcal{P}}, \boldsymbol{\lambda}, \boldsymbol{\mu}]$ with $(\boldsymbol{m}, \boldsymbol{\alpha}, \boldsymbol{\beta})$-center and $(\boldsymbol{E}, \boldsymbol{\alpha}', \boldsymbol{\beta}') -$ axis. Since $\overline{\boldsymbol{f}}$ is intuitionistic fuzzy homology, the center point is not on the axis in the base plane. Suppose that $\overline{\boldsymbol{f}}$ is an intuitionistic fuzzy collineation different from the intuitionistic fuzzy unit collineation. The center $(\boldsymbol{m}, \boldsymbol{\alpha}, \boldsymbol{\beta})$ and each point on the axis $(\boldsymbol{E}, \boldsymbol{\alpha}', \boldsymbol{\beta}')$ are invariant. All points of Intuitionistic Fuzzy Fano plane are on the intuitionistic fuzzy lines passing through the center and all of these intuitionistic fuzzy lines intersect with the axis. So the center and the intersection point on the axis are invariant under $\overline{\boldsymbol{f}}$. There remains only one intuitionistic fuzzy point on the intuitionistic fuzzy line passing through $(\boldsymbol{m}, \boldsymbol{\alpha}, \boldsymbol{\beta})$ such that its imagine is undefined. Since the center, the point and its image are intuitionistic fuzzy collinear, this point has to turn into itself under $\overline{\boldsymbol{f}}$. In this case, each point remains invariant. So $\overline{\boldsymbol{f}}$ is the intuitionistic fuzzy unit collineation.

Theorem 4.3. Intuitionistic Fuzzy Fano plane has only one central intuitionistic fuzzy collineation different from the unit collineation.

Proof. Let $\overline{f}$ be a central intuitionistic fuzzy collineation of Intuitionistic Fuzzy Fano plane different from the unit collineation. From previous theorem, if $\overline{f}$ is an intuitionistic fuzzy homology, then $\overline{f}$ is the intuitionistic fuzzy unit collineation. Suppose that $\overline{f}$ is an intuitionistic fuzzy elation. The center is on the axis. The image of any intuitionistic fuzzy point not on the axis must be on an intuitionistic fuzzy line joining it to the center. Since $\overline{f}$ is different from the intuitionistic fuzzy unit collineation, there are two remaining intuitionistic fuzzy points on this line which must match each other under $\overline{f}$. So there is only one central intuitionistic fuzzy collineation different from the unit collineation in Intuitionistic Fuzzy Fano projective plane.

Corallary Intuitionistic Fuzzy Fano plane has only one intuitionistic fuzzy elation different from the unit collineation.

From now on, we considered the intuitionistic fuzzy projective plane $[\mathcal{P}, \lambda, \mu]$ with the base plane $\mathcal{P}$ and (λ, μ) in the following form:

$$
\begin{array}{llll}
(\lambda, \mu): & \mathcal{P}G(V) & \rightarrow & [0,1] \times [0,1] \\
 & q & \rightarrow & (a_0, b_0) \\
 & p & \rightarrow & (a_1, b_1), \quad p \in L \backslash \{q\} \\
 & p & \rightarrow & (a_2, b_2), \quad p \in \mathcal{P}G(V) \backslash \{L\}
\end{array}
$$

where $\boldsymbol{L}$ is an intuitionistic projective line of $\boldsymbol{\mathcal{P}}$ containing $\boldsymbol{q}$ and $\boldsymbol{a_0} \geq \boldsymbol{a_1} \geq \boldsymbol{a_2}$, $\boldsymbol{b_0} \leq \boldsymbol{b_1} \leq \boldsymbol{b_2}, \boldsymbol{0} \leq \boldsymbol{a_i} + \boldsymbol{b_i} \leq \boldsymbol{1}$, i=0, 1, 2.

The intuitionistic fuzzy point $(\boldsymbol{q}, \boldsymbol{a_0}, \boldsymbol{b_0})$ and the intuitionistic fuzzy line $(\boldsymbol{L}, \boldsymbol{a_1}, \boldsymbol{b_1})$ are called as the base point, the base line of the intuitionistic fuzzy projective plane $[\boldsymbol{\mathcal{P}}, \boldsymbol{\lambda}, \boldsymbol{\mu}]$, respectively. Some properties under the central intuitionistic fuzzy collineation in $[\boldsymbol{\mathcal{P}}, \boldsymbol{\lambda}, \boldsymbol{\mu}]$ depending on the base line, the base point and the membership degrees of $[\boldsymbol{\mathcal{P}}, \boldsymbol{\lambda}, \boldsymbol{\mu}]$ are introduced with the following theorems.

Theorem 4.4. Suppose that $\overline{\boldsymbol{f}}$ is a central intuitionistic fuzzy collineation of $[\boldsymbol{\mathcal{P}}, \boldsymbol{\lambda}, \boldsymbol{\mu}]$ defined by the collineation f of the base plane $\boldsymbol{\mathcal{P}}$ with $\boldsymbol{m}$-center and $\boldsymbol{E}$ $-$axis.

i) The center $(\boldsymbol{m}, \boldsymbol{\alpha}, \boldsymbol{\beta})$ is invariant under the central intuitionistic fuzzy collineation $\overline{\boldsymbol{f}}$.

ii) The axis $(\boldsymbol{E}, \boldsymbol{\alpha}', \boldsymbol{\beta}')$ is invariant under the central intuitionistic fuzzy collineation $\overline{\boldsymbol{f}}$.

Proof. i) Let $(\boldsymbol{m}, \boldsymbol{\alpha}, \boldsymbol{\beta})$ be the center of $\overline{\boldsymbol{f}}$ in $[\boldsymbol{\mathcal{P}}, \boldsymbol{\lambda}, \boldsymbol{\mu}]$. The lines $\boldsymbol{L_i}$, $\boldsymbol{i} = \boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{n} + \boldsymbol{1}$ passing through the center are invariant from the definition of the center of $\boldsymbol{f}$ in $\boldsymbol{\mathcal{P}}$. Let the center $(\boldsymbol{m}, \boldsymbol{\alpha}, \boldsymbol{\beta})$ be the intersection point of the intuitionistic fuzzy lines $(\boldsymbol{L_i}, \boldsymbol{\alpha_i}, \boldsymbol{\beta_i})$, $\boldsymbol{i} = \boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{n} + \boldsymbol{1}$ in $[\boldsymbol{\mathcal{P}}, \boldsymbol{\lambda}, \boldsymbol{\mu}]$.

$$(\boldsymbol{m}, \boldsymbol{\alpha}, \boldsymbol{\beta}) = \bigcap_{i=1}^{n+1} (L_i, \alpha_i, \beta_i) = \left(\bigcap_{i=1}^{n+1} L_i, \bigwedge_{i=1}^{n+1} \alpha_i, \bigvee_{i=1}^{n+1} \beta_i \right)$$

By using the definition $\bar{f}$,

$$\begin{aligned} \bar{f}(m, \alpha, \beta) &= (f(m), \alpha, \beta) \\ &= \left(f\left(\bigcap_{i=1}^{n+1} L_i \right), \bigwedge_{i=1}^{n+1} \alpha_i, \bigvee_{i=1}^{n+1} \beta_i \right) \\ &= \left(\bigcap_{i=1}^{n+1} f(L_i), \alpha, \beta \right) \\ &= \left(\bigcap_{i=1}^{n+1} L_i, \alpha, \beta \right) = (m, \alpha, \beta) \end{aligned}$$

is obtained. So the center (m, α, β) is invariant under $\bar{f}$.

ii) For all $(p_i, \alpha_i, \beta_i), i = 1, 2, \dots, n$ on the axis, from the definition of the axis of the intuitionistic fuzzy collineation $\bar{f}$,

$$\bar{f}(p_i, \alpha_i, \beta_i) = (p_i, \alpha_i, \beta_i), i = 1, 2, \dots, n+1.$$

The axis E can written $E = \cup_{i=1}^{n+1} p_i$ in $\mathcal{P}$ and

$(E, \alpha', \beta') = \langle \cup_{i=1}^{n+1} p_i, \wedge_{i=1}^{n+1} \alpha'_i, \vee_{i=1}^{n+1} \beta'_i \rangle$ in $[\mathcal{P}, \lambda, \mu]$.

By using the definition of the intuitionistic fuzzy collineation $\bar{f}$ in $[\mathcal{P}, \lambda, \mu]$.

$$\begin{aligned} \bar{f}(E, \alpha', \beta') &= \bar{f}\left(\langle \bigcup_{i=1}^{n+1} p_i, \bigwedge_{i=1}^{n+1} \alpha'_i, \bigvee_{i=1}^{n+1} \beta'_i \rangle \right) \\ &= \langle f\left(\bigcup_{i=1}^{n+1} p_i \right), \bigwedge_{i=1}^{n+1} \alpha'_i, \bigvee_{i=1}^{n+1} \beta'_i \rangle \\ &= \langle \bigcup_{i=1}^{n+1} f(p_i), \bigwedge_{i=1}^{n+1} \alpha'_i, \bigvee_{i=1}^{n+1} \beta'_i \rangle \\ &= \langle \bigcup_{i=1}^{n+1} p_i, \bigwedge_{i=1}^{n+1} \alpha'_i, \bigvee_{i=1}^{n+1} \beta'_i \rangle \\ &= \left(E, \bigwedge_{i=1}^{n+1} \alpha'_i, \bigvee_{i=1}^{n+1} \beta'_i \right) = (E, \alpha', \beta') \end{aligned}$$

is obtained. So the axis (E, α', β') is invariant under $\bar{f}$.

Theorem 4.5. Suppose that $\bar{f}$ is a central intuitionistic fuzzy collineation with (m, α, β) −center of $[\mathcal{P}, \lambda, \mu]$. If $\bar{f}$ leaves two distinct lines that do not pass through the center invariant, then $\bar{f}$ is the intuitionistic fuzzy unit collineation.

Proof. Let (L_1, α_1, β_1) and (L_2, α_2, β_2) be two intuitionistic fuzzy invariant line that do not pass through the center of $\bar{f}$ in $[\mathcal{P}, \lambda, \mu]$. From [3], the intersection point of these lines remains invariant under $\bar{f}$. Since the center and every intuitionistic fuzzy line passing through the center are invariant under $\bar{f}$, the intersection points of the intuitionistic fuzzy lines (L_1, α_1, β_1) and (L_2, α_2, β_2) with the lines passing through the center (m, α, β) are invariant under $\bar{f}$. So, every point on (L_i, α_i, β_i), $i = 1, 2$ is invariant and it is well shown that if two intuitionistic fuzzy points that are not on an intuitionistic fuzzy line which is pointwise are invariant, $\bar{f}$ is the unit in [3]. So the proof is completed.

Theorem 4.6. Suppose that $\bar{f}$ is an intuitionistic fuzzy collineation of $[\mathcal{P}, \lambda, \mu]$ defined by the collineation f of tha base plane $\mathcal{P}$. Then,

i) Let f be a homology such that its center m is the base point on the base line L. If the intuitionistic fuzzy collineation $\bar{f}$ with the center (q, a_0, b_0) has an axis, then $a_1 = a_2$, $b_1 = b_2$.

ii) Let f be a homology such that its center m is on the base line L different from the base point. If the intuitionistic fuzzy collineation $\bar{f}$ with the center (m, a_1, b_1) has an axis, then there are the relationships $a_1 = a_2$, $b_1 = b_2$ or $a_0 = a_1 = a_2$, $b_0 = b_1 = b_2$ among the memberhip degrees of $[\mathcal{P}, \lambda, \mu]$.

iii) Let f be a homology such that its center m not on the base line L. If the intuitionistic fuzzy collineation $\bar{f}$ has an axis (E, a_2, b_2), then there are the relationships $a_1 = a_2$, $b_1 = b_2$ or $a_0 = a_1 = a_2$, $b_0 = b_1 = b_2$ in $[\mathcal{P}, \lambda, \mu]$.

Proof. i) Let f be (q, E)-homology. Since the center q is not on the axis, $L \neq E$.

Let the intuitionistic fuzzy collineation $\bar{f}$ with the center (q, a_0, b_0) have an axis. Suppose that the axis E has the membership and nonmembership degree (a_2, b_2) different from (a_1, b_1). Let's take (p_1, a_1, b_1) and (p_2, a_2, b_2) be on the axis (E, a_2, b_2) such that $p_1 = qp_1 \cap E$.

$$\begin{aligned} \bar{f}(p_1, a_1, b_1) &= \bar{f}\left((qp_1, a_1, b_1) \cap (E, a_2, b_2) \right) \\ &= (qp_1 \cap E, a_2, b_2) = (p_1, a_2, b_2). \end{aligned}$$

Since (E, a_2, b_2) is the axis of the intuitionistic fuzzy collineation $\bar{f}$, (p_1, a_1, b_1) must be invariant. In this case $a_1 = a_2$ and $b_1 = b_2$ are obtained.

ii) Let f be (m, E)-homology such that $m \circ L$, $m \neq q$.

Case 1: Let the axis E intersect the base line on a point different from the base point q.

Suppose that the axis of $\bar{f}$ is (E, a_2, b_2).

Let's take the intuitionistic fuzzy points (p_1, a_1, b_1) and (p_2, a_2, b_2) be on the axis (E, a_2, b_2) such that $p_1 = mp_1 \cap E$. Then, $\bar{f}(p_1, a_1, b_1)$ is $\bar{f}((mp_1, a_1, b_1) \cap (E, a_2, b_2))$=$(f(mp_1 \cap E), a_1 \wedge a_2, b_1 \vee b_2)$=$(p_1, a_2, b_2)$. Since (p_1, a_1, b_1) is on the axis, it is invariant. Hence, $a_1 = a_2$, $b_1 = b_2$ are obtained.

Case 2: Let the axis E of f intersect the base line L on the base point q.

Let's take the intuitionistic fuzzy points (q, a_0, b_0) and (p, a_2, b_2) be on the axis (E, a_2, b_2) such that $q = mq \cap E$. Then, $\bar{f}\,(q, a_0, b_0)$ is $\bar{f}\left((mq, a_1, b_1) \cap (E, a_2, b_2)\right)$=$(f(mq \cap E), a_1 \wedge a_2, b_1 \vee b_2)$=$(mq \cap E, a_2, b_2) = (q, a_2, b_2)$. Since (q, a_0) is on the axis, so it is intuitionistic fuzzy invariant. $a_0 = a_2$ and $b_0 = b_2$ are obtained.

From the relationships $a_0 \geq a_1 \geq a_2$ and $b_0 \leq b_1 \leq b_2$, among the membership degrees in $[\mathcal{P}, \lambda]$, $a_0 = a_1 = a_2$ and $b_0 = b_1 = b_2$ are obtained.

iii) Suppose that f is (m, E)- homology, $m \not\in L$. Two results are obtained. The axis can be the base line (L, a_1, b_1) or different from the base line.

If the axis (E, a_2, b_1) of $\bar{f}$ is the base line (L, a_1, b_1), it is easily seen that $a_0 = a_1 = a_2, b_0 = b_1 = b_2$.

When the axis (E, a_2, b_2) of $\bar{f}$ is different from the base line (L, a_1, b_1), it intersects the base line in the base point (q, a_0, b_0) or a point (p, a_1, b_1) on the base line (L, a_1, b_1). If the intersection point is the base point (q, a_0, b_0), the relationships $a_0 = a_1 = a_2$ and $b_0 = b_1 = b_2$ are obtained easily. Similarly, if the intersection point is different from the base point (q, a_0, b_0) on the base line, the equations $a_1 = a_2$ and $b_1 = b_2$ are obtained.

Corallary Suppose that $\bar{f}$ is a central intuitionistic fuzzy collineation of $[\mathcal{P}, \lambda, \mu]$ defined by the (m, E) −homology f of the base plane $\mathcal{P}$ such that the base point q is on the axis E, then there is only one membership degree in $[\mathcal{P}, \lambda, \mu]$.

Theorem 4.7. Suppose that the collineation f of the base plane $\mathcal{P}$ is (m, E)−central collineation. Then, the intuitionistic fuzzy collineation $\bar{f}$ of $[\mathcal{P}, \lambda, \mu]$ defined by the collineation of f is the central collineation, too.

Proof. Let f be (m, E)−central collineation of the base plane $\mathcal{P}$. Since E is the axis of f, E is pointwise. From the definition of the intuitionistic fuzzy collineation $\bar{f}$, the intuitionistic fuzzy line (E, α', β') is pointwise. So the intuitionistic fuzzy line (E, α', β') is the axis for the intuitionistic fuzzy collineation $\bar{f}$. Similarly, since m is the axis of f, m is linewise. Using the definition $\bar{f}$, $\bar{f}\,(m, \alpha, \beta) = (f(m), \alpha, \beta) = (m, \alpha, \beta)$ and for any intuitionistic fuzzy line $(L_i, \alpha_i', \beta_i'), i = 1, 2, \ldots, n+1$ passing through (m, α, β), $\bar{f}\,(L_i, \alpha_i', \beta_i'), = (f(L_i), \alpha_i', \beta_i') = (L_i, \alpha_i', \beta_i')$. Hence the intuitionistic fuzzy point (m, α, β) is the center for the intuitionistic fuzzy collineation $\bar{f}$.

Consequently, the intuitionistic fuzzy collineation $\bar{f}$ is $\left((m, \alpha), (E, \beta)\right)$ − central intuitionistic fuzzy collineation.

Corollary There is a central intuitionistic fuzzy collineation determined by the central collineation f of the base plane $\mathcal{P}$ in the intuitionistic fuzzy projective plane $[\mathcal{P}, \lambda, \mu]$.

5. Conclusion

The fuzzy counterparts and intuitionistic fuzzy counterparts of the central collineations defined in classical projective planes are introduced in fuzzy and intuitionistic fuzzy projective planes, respectively. It is determined that every homology of both Fuzzy Fano plane and Intuitionistic Fuzzy Fano plane is an unit collineation and also Fuzzy Fano projective plane and Intuitionistic Fuzzy Fano projective plane have only one elation distinct from the unit collineation.

It is seen that if the axis of a fuzzy homology of the fuzzy projective plane is passing through the base point, then all membership degrees in the fuzzy projective plane must be equal. The same result is obtained for the intuitionistic fuzzy projective plane. That is, in this case the fuzzy and intuitionistic fuzzy projective planes are crisp.

Consequently, the obtained results and introduced theorems showed that there is an important relationship between the axis, the center of the central collineations in the fuzzy and intuitionistic fuzzy projective plane and the base point, the base line of the fuzzy and intuitionistic fuzzy projective plane.

These obtained results on the central fuzzy collineation and the central intuitionistic fuzzy collineations have an important effect on enriching the theory of the fuzzy and intuitionistic fuzzy geometries.

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