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Research Article

On the Solution of a Nonhomogeneous Fisher-Kolmogorov Equation

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Abstract

In this paper, as a new method for obtaining the numerical solution of nonhomogeneous Fisher-Kolmogorov equation, Bernoulli-collocation method is introduced. Bernoulli-collocation method is employed for three different cases of Fisher-Kolmogorov equation. Obtained numerical results are presented in the tables and graphicals forms.

Keywords: Bernoulli collocation method, Fisher Equation, Numerical Solution

Homojen Olmayan Fisher-Kolmogorov Denkleminin Çözümü Üzerine

Öz

Bu makalede, homojen olmayan Fisher-Kolmogorov denkleminin sayısal çözümünü elde etmek için yeni bir yöntem olarak Bernoulli sıralama yöntemi tanıtılmaktadır. Fisher-Kolmogorov denkleminin üç farklı durumu için Bernoulli sıralama yöntemi kullanılmıştır. Elde edilen sayısal sonuçlar tablolar ve grafik formlarda sunulmuştur.

Anahtar Kelimeler: Bernoulli sıralama yöntemi, Fisher Denklemi, Nümerik çözüm

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1. Introduction

Generally problems existing in the engineering and nature are modeled by means of differential equations. After modeling the problem in the form of differential equation, main important question arises. Does the model have a solution? If yes, What is the solution? If the differential equation corresponding to problem is linear, the solution of this differential equation can be generally found. If the differential equation is nonlinear, finding the solution of this differential equation is not easy and sometimes it is impossible or unthinkable. In that cases, finding the numerical solutions of nonlinear differential equation is considered. One of the nonlinear differential equations is Fisher-Kolmogorov equation and Fisher-Kolmogorov equation was introduced in (Adomian, 1995) for decsribing the reaction-diffusion phonema in the chemical sciences. Several numerical methods are introduced for obtaining the numerical solution of Fisher-Kolmogorov equation. In (Sweilam, ElSakout & Muttardi, 2021), authors derived a new compact finite difference scheme in the spatial direction and used the semi-implicit Euler-Maruyama approach in the temporal direction to study a stochastic extended FisherKolmogorov equation with multiplicative noise numerically. In (Kadri & Omrani, 2011), a Crank–Nicolson type finite difference scheme to approximate the nonlinear evolutionary Extended Fisher–Kolmogorov (EFK) equation is presented. In (Araujo, 2014), authors considered an existence result for periodic solutions for a class of fourth-order ordinary differential equations involving extended Fisher–Kolmogorov and Swift–Hohenberg equations, where under a suitable growth condition on the nonlinear term, one proves an existence result by applying Mawhin's continuation theorem. In (Cabada, Souroujon & Tersian, 2012), existence of heteroclinic solutions for semilinear second-order difference equations related to the Fisher–Kolmogorov's equation is presented. In (Andreu, Caselles & Maz´on, 2010), a Fisher–Kolmogorov type equation is taken into account and it is proved that the existence and uniqueness of finite speed moving fronts and the existence of some explicit solutions in a particular regime of the equation. In (Yeun, 2013), it is studied the extended Fisher–Kolmogorov (EFK) equation and its variants. In (Danumjaya & Pani, 2005), a second-order splitting combined with orthogonal cubic spline collocation method is formulated and analysed for the extended Fisher–Kolmogorov equation. With the help of Lyapunov functional, a bound in maximum norm is derived for the semidiscrete solution. Optimal error estimates are established for the semidiscrete case. Specifically, in this paper, a nonhomogeneous Fisher-Kolmogorov is presented. Bernoulli collocation method is recalled and adopted to Fisher-Kolmogorov equation. By means of mathematical software, numerical solution of Fisher-Kolmogorov equation is obtained. Obtained numerical solution is plotted for different values of v. Also, error analysis is presented by means of table. Let us take into account following nonhomogeneous Fisher-Kolmogorov equation as follows;

$$\frac{\partial w(t,x)}{\partial t}+v\frac{\partial^4 w(t,x)}{\partial x^4}-\frac{\partial^2 w(t,x)}{\partial x^2}+\psi(w)=f(t,x) \qquad (1)$$

in which $w(t,x)$ is reaction-diffusion function, $(x,t)\in[0,\ell]\times[0,T]$, $\psi(w)=w^3-w$, $f(t,x)$ is external function effected on the reaction-diffusion and v is a non negative constant. Eq.(1) is subject to the following boundary conditions;

$$w(t,x)=0,\quad w_{xx}(t,x)=0 \quad at \quad x=0,\ell \qquad (2)$$

and following initial conditions;

$$w(t,x)=w_0(x) \quad at \quad t=0. \qquad (3)$$

2. Bernoulli collocation method

The recurrence relation of the Bernoulli polynomials is defined by the following relation;

$$B_n(x)=2xB_{n-1}(x)+B_{n-2}(x) \qquad (4)$$

For $n\geq 3$,

$$B_1(x)=1, \qquad B_2(x)=2x\,. \qquad (5)$$

The first few Bernoulli polynomials are

$$\begin{aligned} B_1(x) &= 1,\\ B_2(x) &= x-\frac{1}{2},\\ B_3(x) &= x^2-x-\frac{1}{6},\\ B_4(x) &= x^3-\frac{3}{2}x^2+\frac{x}{2},\\ &\ \ \vdots \end{aligned} \qquad (6)$$

Our goal is to get the approximate solution as the truncated Bernoulli series defined by

$$y(x)=\sum_{n=1}^{N+1} c_n\,B_n(x) \qquad (7)$$

where $B_n(x)$ denotes the Bernoulli polynomials; c_n $(1\leq n\leq N+1)$ are the unknown coefficients for Bernoulli polynomial, and N is any positive integer which possess $N\geq m.$ Let us assume that linear combination of Bernoulli polynomials Eq.(7) is an approximate solution of Eq.(1). Our purpose is to determine the matrix forms of Eq.(1) by using (7). Firstly, we can write Bernoulli polynomials (5) in the matrix form

$$\boldsymbol{B}(x)=\boldsymbol{T}(x)\,\boldsymbol{M} \qquad (8)$$

where $\boldsymbol{B}(x)=[B_1(x)\ B_2(x)\ ...\ B_{N+1}(x)]$, $\boldsymbol{T}(x)=(1\ x\ x^2\ x^3\ ....x^N)$, $\boldsymbol{C}=(c_1\ \ c_2\ \ ...\ \ c_{N+1})^T\ and$

$$M=\begin{pmatrix}1 & -\frac{1}{2} & \frac{1}{6} & 0 & -\frac{1}{30} & 0 & \frac{1}{42} & 0 & -\frac{1}{30}\\ 0 & 1 & -1 & \frac{1}{2} & 0 & -\frac{1}{6} & 0 & \frac{1}{6} & 0\\ 0 & 0 & 1 & -\frac{3}{2} & 1 & 0 & -\frac{1}{2} & 0 & \frac{2}{3}\\ 0 & 0 & 0 & 1 & -2 & \frac{5}{3} & 0 & -\frac{7}{6} & 0\\ 0 & 0 & 0 & 0 & 1 & -\frac{5}{2} & \frac{5}{2} & 0 & -\frac{7}{3}\\ 0 & 0 & 0 & 0 & 0 & 1 & -3 & \frac{7}{2} & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{7}{2} & \frac{14}{3}\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -4\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\end{pmatrix}$$

The matrix form of Eq.(7) by a truncated Bernoulli series is given by

$$\mathcal{Y}(x)=\boldsymbol{B}(x)\,\boldsymbol{C}. \tag{9}$$

By using Eq.(8) and Eq.(9), the matrix relation is expressed as

$$\begin{aligned}\mathcal{Y}(x)\ &\cong\ \mathcal{Y}_N(x)=\boldsymbol{T}(x)\boldsymbol{MC}\\ \mathcal{Y}^{(\gamma)}(x)\ &\cong\ \mathcal{Y}_N^{(\gamma)}(x)\\ &=\boldsymbol{T}(x)\boldsymbol{X}_{(\gamma)}^{(x)}\boldsymbol{D}_{(\gamma)}\boldsymbol{MC}\\ \mathcal{Y}''(x)\ &\cong\ \mathcal{Y}_N''(x)=\boldsymbol{T}(x)\boldsymbol{D}^2\boldsymbol{MC}\end{aligned} \tag{10}$$

where

$$\boldsymbol{X}_{(\gamma)}(x)=[0,x^{1-\gamma},x^{2-\gamma},\dots,x^{N-\gamma}]$$

$$\boldsymbol{D}=\begin{bmatrix}0&1&0&0&0&0&\cdots&0\\0&0&2&0&0&0&\cdots&0\\0&0&0&3&0&0&\cdots&0\\0&0&0&0&4&0&\cdots&0\\0&0&0&0&0&5&\cdots&0\\0&0&0&0&0&0&\cdots&0\\\vdots&\vdots&\vdots&\vdots&\vdots&\ddots&\ddots&N\\0&0&0&0&0&0&\cdots&0\end{bmatrix}$$

$$\boldsymbol{D}^0=\begin{bmatrix}1&0&0&0&0&0&\cdots&0\\0&1&0&0&0&0&\cdots&0\\0&0&1&0&0&0&\cdots&0\\0&0&0&1&0&0&\cdots&0\\0&0&0&0&1&0&\cdots&0\\0&0&0&0&0&1&\cdots&0\\\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\ddots&0\\0&0&0&0&0&0&\cdots&1\end{bmatrix}$$

$$\boldsymbol{T}\ =\begin{bmatrix}\boldsymbol{T}(x_0)\\\boldsymbol{T}(x_1)\\\vdots\\\boldsymbol{T}(x_N)\end{bmatrix}=\begin{bmatrix}1&x_0&\cdots&x_0^N\\1&x_1&\cdots&x_1^N\\1&\vdots&\cdots&\vdots\\1&x_N&\cdots&x_N^N\end{bmatrix}$$

$$\boldsymbol{D}_{(\gamma)}=\begin{bmatrix}0&0&0&\dots&0\\0&\frac{\Gamma(2)}{\Gamma(2-\gamma)}&0&\dots&0\\0&0&\frac{\Gamma(3)}{\Gamma(3-\gamma)}&\dots&0\\\vdots&\vdots&\vdots&\ddots&\vdots\\0&0&0&\dots&\frac{\Gamma(N)}{\Gamma(N-\gamma)}\end{bmatrix}$$

By using Eq.(10), we obtain the following relation

$$\boldsymbol{Y}^{(k)}(x)=\boldsymbol{T}(x)\boldsymbol{D}^k\boldsymbol{MC} \tag{11}$$

By substituting the Bernoulli collocation points given by

$$x_i=a+\frac{(b-a)i}{N}\,,i=0,1,\dots N \tag{12}$$

into Eq. (11), we obtain

$$\boldsymbol{Y}^{(k)}(x_i)=\boldsymbol{T}(x_i)\boldsymbol{D}^k\boldsymbol{MC}\,,\ \ k=0,\gamma,2. \tag{13}$$

and the compact form of the relation (13) becomes

$$\boldsymbol{Y}^{(k)}=\boldsymbol{TD}^k\boldsymbol{MC},\ \ k=0,\gamma,2. \tag{14}$$

In this way, the unknown Bernoulli coefficients c_n, $n=1,2,\dots,N+1$ are obtained by solving the system. Then, these coefficients are substituted into (7), and the approximate solution is obtained.

3. Simulation results and discussions

In this section, obtained numerical solutions via Bernoulli collocation method are presented in the table and graphical forms. The $f(t,x)$ is taken into account as xe^{-t}. Also, l and T are considered as 1 and 1, respectively. Numerical results show that introduced new numerical method for solving Fisher-Kolmogorov is very effective. In table 1, numerical solution of the equation under consideration is given at the some points of t in [0,1] for different values of υ. In table 2, error analysis of the numerical solution gained by means of Bernoulli collocation method is presented and observations on table 2 reveals that Bernoulli collocation method has the very good accurate for obtaining solutions of nonlinear equations. In Figure 1, obtained numerical solutions is illustrated for (x,t) in $[0,1]\times[0,1]$.

Table 1. Some values of numerical solutions for different t at $x = 0.5$.

v \ t	0.0	0.2	0.4	0.6	0.8	1.0
0.1	0.0	0.0034678	0.0147922	0.0256025	0.0250025	0.0069034
0.001	0.0	0.0048285	0.0243134	0.0471310	0.0523136	0.0249079
0.0001	0.0	0.0048450	0.0244505	0.0474783	0.0528039	0.0252991

Table 2. Error analysis of numerical solutions obtained by Bernoulli Collocation method for different t at $x = 0.5$

v \ t	0.0	0.2	0.4	0.6	0.8	1.0
0.1	0.0	3.60822×10^{-16}	1.58207×10^{-15}	1.33227×10^{-14}	2.13163×10^{-14}	8.15554×10^{-3}
0.001	0.0	4.85723×10^{-17}	3.60822×10^{-16}	4.44089×10^{-16}	2.66454×10^{-15}	3.67326×10^{-4}
0.0001	0.0	2.77556×10^{-17}	8.32667×10^{-17}	5.55112×10^{-16}	6.21725×10^{-15}	3.42666×10^{-4}

6. Conclusions

In this paper, numerical solution of nonhomogeneous Fisher-Kolmogorov equation is obtanied by employing the Bernoulli-Collocation method. Obtained results are presented in tables and graphical forms. After observing the tables and graphic for numerical solutions obtained by Bernoulli collocation method, it reveals that Bernoulli collocation method is very effective and it is robust method for obtaining numerical solutions of other nonlinear equations.

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