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Research Article

V_{S30}-based Coherency Model: Istanbul case

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Abstract

Strong ground motion caused by earthquakes at every point of extended structures would not be same. This difference in ground movement has an important effect on the design of these types of structures. Meanwhile, the seismic resistant design has been lead to investigate the variability of earthquake ground motion over last decades. In this study, frequency domained variability named coherency is considered. Several coherency models have been proposed without considering soil effect. In this context, spatial variation of seismic ground motion based on the average shear wave velocity over the upper 30 m of depth, V_{S30} is analyzed. Initially, coherency values are calculated using data triggered during six earthquakes recorded by the Istanbul Earthquake Rapid Response System. Lagged coherency data is considered in the process to get the coherency model. Nonlinear regression analysis is used for the model to obtain a good-fit to observed data. A coefficient is defined based on V_{S30} values of the station-pairs. The cohereny model based on this coefficient of V_{S30} is derived for EW and NS components. It is expected that coherency function decreases with the increase of frequency and separation distance. The decrease in the coefficient of V_{S30} causes decrease in coherency. The reason is that the heterogenity in soil causes the scattering of the earthuqke waves. The variance in the coherency model between EW and NS components is small. This coherency model is used to simulate spatial variable ground motion for the accurate seismic design of extended structures for the future studies.

Keywords: Earthquakes, Istanbul Earthquake Rapid Response System, Coherency Model, V_{S30}.

V_{S30} değerine bağlı koherans modeli: İstanbul modeli

Öz

Depremlerin yol açtığı kuvvetli yer hareketi uzun yapıların her yerinde aynı olmayacaktır. Yer hareketindeki bu farklılığın, uzun yapıların tasarımı üzerinde önemli bir etkisi vardır. Depreme dayanıklı tasarımın, son yüzyıllarda deprem yer hareketinin değişkenliğini araştırmada etkisi olmuştur. Bu çalışmada, koherans adı verilen frekans tanım alanı yönünden deprem yer hareketlerinin değişkenliği ele alınmıştır. Bugüne kadar genelde, zemin etkisi dikkate alınmadan çeşitli koherans modelleri oluşturulmuştur. Bu bağlamda, 30 m derinliğin üstündeki ortalama kayma dalgası hızına (V_{S30}) bağlı olarak deprem yer hareketinin mekansal değişimi analiz edilmiştir. İlk olarak, koherans değerleri İstanbul Deprem Acil Müdahale Sistemi tarafından kaydedilen altı depremin verileri kullanılarak hesaplanmıştır. Koherans modelini elde etmek için duraklamalı koherans verileri dikkate alınmıştır. Modelin kayıtlı verilerde en iyi sağlaması için doğrusal olmayan regresyon analizi kullanılmıştır. İkili istasyon gruplarının V_{S30} değerlerine dayanarak bir katsayı tanımlanmıştır. Bu V_{S30} katsayısına bağlı koherans modeli; EW, NS ve dikey bileşenler için oluşturulmuştur. Beklendiği üzere, frekans ve istasyonlar arası mesafesinin artmasıyla koherans fonksiyonunun azaldığı gözlenmiştir. V_{S30} katsayısındaki azalma, koherans değerlerinde azalmaya neden olmuştur. Bunun nedeni, zemindeki heterojenliğin deprem dalgalarında saçılıma neden olduğudur. EW ve NS bileşenleri için üretilen koherans modelleri arasındaki fark oldukça küçüktür. Düşey bileşen için üretilen model yatay için üretilenden farklıdır. Gelecekteki çalışmalarda, elde edilen koherans modeli mekansal geniş yapıların depreme dayanıklı tasarımı için mekansal değişen yer hareketlerini simüle etmek için kullanılır.

Anahtar Kelimeler: Depremler, İstanbul Deprem Acil Müdahale Sistemi, Koherans Modeli, V_{S30}.

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1. Introduction

One of the most significant natural disasters affecting people is earthquakes especially considering fatality (Dilmaç and Demir, 2019). A very important feature of earthquake loads on extended structures such as bridges and buried pipelines is the spatial variability of seismic ground motion (SVGM). Several parameters are used to define this variability: Fourier amplitude spectra, peak ground acceleration (Bayrak, 2019), peak ground velocity, pseudo-velocity response spectrum. Additionally, coherency in frequency domain is used to describe the spatial variability. Abrahamson (1993), Harichandran and Vanmarcke (1986), Harichandran (1988), Harichandran (1991), Loh and Yeh (1988), Loh and Lin (1990), Novak (1987), Oliveira et al. (1991), Ramadan and Novak (1993), Zerva and Zhang (1997) and Cacciola and Deodatis (2011) proposed coehermcy models. Zerva and Zervas (2002) and Zerva (2009) reviewed general properties of spatial variation of earthquake ground motion. Harmandar et al. (2006a, 2006b) studied on the statistical properties of spatial variability of ground motion data of two earthquakes recorded by Istanbul Earthquake Rapid Response System (IERRS). Harmandar et al. (2012) developed a new methodology for the interpolation of peak ground acceleration based on the spatial distribution of discrete array stations using data from IERRS.

As aforementioned, importance of spatial variability in modelling of earthquake ground motion is known for the design of above or under ground structures and systems where multiple-support excitation needs to be considered. Several methods have been used for the derivation of spatial variability. Spectral representation method (Rice, 1944;Shinozuka, 1972); auto-regressive, moving-average, and auto-regressive-moving-average models (Conte et al., 1992; Ellis and Cakmak, 1991; Mignolet and Spanos, 1992); local average subdivision method (Fenton, 1990) and the covariance matrix decomposition (Hao et al., 1989; Zerva and Katafygiotis, 2000) are some of the methods used for the simulation of spatially variable strong ground motion. Additionally, Abrahamson (1992) studied envelope functions considering random phase variability; Ramadan and Novak (1994) proposed coherency function estimation using a Fourier series. Moreover, Yamamoto (2011) proposed that for the probabilistic assessment of the performance of structures ground motion simulation with appropriate coherency is required.

The reasons for the spatial variation of ground motion are incoherence effect, path effect and local site effect. Incoherence effect is caused by the differences in the amplitudes and phases of earthquake waves. The time delay of the arrival time is the reason for wave passage effect. Local site effect is due to the variance of local soil profiles (Der Kiureghian, 1996). Schneider et al., 1992 studied the effect of the site on SVGM considering the data from rock sites and soil sites. The local site effects on the SVGM have been studied by Zerva and Harada (1997). Abrahamson, 2005 took into account the effect of local site condition on spatial coherency.

In this study, an empirical coherency model is derived considering the average shear wave velocity over the upper 30m of depth (V_{S30}). Data from six earthquakes recorded by Istanbul Earthquake Rapid Response System (IERRS) stations are used in the regression analysis. Furthermore, coherency model is constituted for EW and NS components. The proposed model could be used in the simulation of non-stationary ground motion needed for the design of extended structures.

2. Materials and Method

2.1. Definition of Coherency

Coherency is the variation in Fourier phase and the loss of correlation between two ground motions. It decays generally exponentially in terms of frequency and station separation distances. It defines the degree of similarity of earthquake ground motion data from two stations. It is the ratio between the cross-power spectral density and auto-power spectral density of data taken from separated locations, mathematically. The power spectrum is the Fourier transform of cross covariance function that explains how two separate data are common. The cross power spectrum describes the degree of correlation of two stations under random ground motion.

$$\gamma_{ij}(f,d) = \frac{S_{ij}(f)}{\sqrt{S_{ii}(f)\, S_{jj}(f)}} \tag{1}$$

in which f is frequency, d is separation distance between the stations i and j, $S_{ij}(f)$ is the cross-power spectral density between stations i and j, $S_{ii}(f)$ is the power spectral density at station i and $S_{jj}(f)$ is the power spectral density at station j. Equation (1) calculates the complex form of coherency values. Therefore, the coherency is generally a complex function and can be written as:

$$\gamma_{ij}(f,d) = \left|\gamma_{ij}(f,d)\right| e^{-i\theta_{ij}(f)} \tag{2}$$

in which i in the exponential form denotes the complex number $\sqrt{-1}$ and the phase spectrum is

$$\theta_{ij}(f) = \tan^{-1}\left(\frac{\mathrm{Im}\left|S_{ij}(f)\right|}{\mathrm{Re}\left|S_{ij}(f)\right|}\right) \tag{3}$$

The real part of the coherency function, $\mathrm{Re}\left|S_{ij}(f)\right|$ is commonly referred as unlagged coherency; absolute value of the coherency is named as lagged coherency, $\left|\gamma_{ij}(f,d)\right|$, (Zerva, 2009). The square of the lagged coherency is referred as coherence function, $\left|\gamma_{ij}(f,d)\right|^2$ It is obvious that $0 \le \left|\gamma_{ij}(f,d)\right| \le 1$. Lagged coherency is considered in the regression analysis. Abrahamson et al. (1991) stated that lagged coherency removes the effects of inclined plane wave propagation and generally is used in engineering purposes. Lagged coherency decreases with the increase of frequency and separation distance.

2.2. Data and local site conditions

To ensure the effect of site on SVGM, data from different earthquakes and soil profiles are selected. In this study, six earthquakes registered by the Earthquake Rapid Response System in Istanbul (IERRS) are utilized for the calculation of coherency values. The IERRS was consisting of 100 stations until 2012. After then, 20 stations have been added. Detailed information of IERRS and data can be found in Harmandar et al. (2012). The epicenters of earthquakes utilized in the present work are shown in Figure 1. General properties of the chosen events are summarized in Table 1.

For the generation of coherency values, acceleration data are baseline-corrected and filtered with a butter-worth bandpass filter (4th order). To determine the filter range for the elimination of noise from real earthquake data, Fourier amplitude spectrum and signal to noise ratio are utilized. S-wave window lengths are identified by inspection for each record and a five per cent cosine tapering is applied. After preprocessing and alignment operations, the coherency values are obtained by calculating the power spectral densities and cross-spectral density. Additionally, determination of smoothing windows is essential in the coherency procedure. An 11-point Hamming window is suggested when the data length is less than 2000 steps for the engineering purposes (Abrahamson et al., 1991). Therefore, in this study, coherency values are determined by using 11-point Hamming window for EW and NS components of earthquake ground motion data.

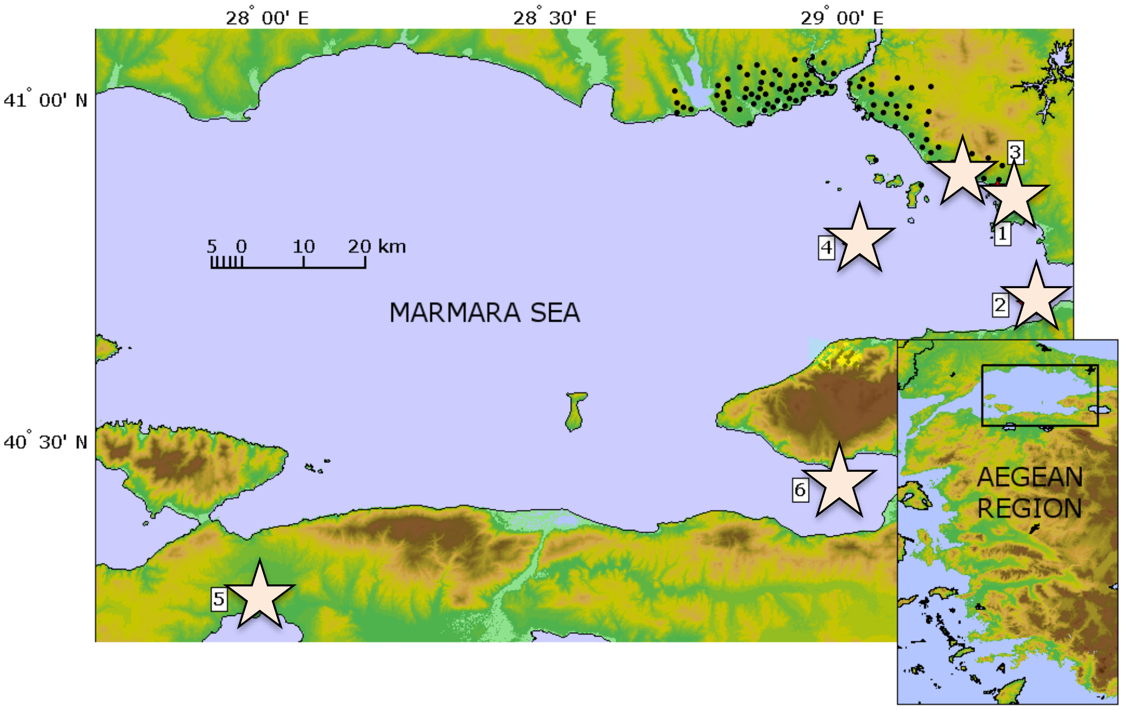

Figure 1. Epicenters of the selected earthquakes recorded by the Istanbul Earthquake Rapid Response System. Stars are the epicenters of the earthquake used in the regression analysis. Numbers correspond to the earthquakes mentioned in Table 1 (after Harmandar et al., 2012)

Table 1. Source properties of the earthquakes registered by IERRS (http://www.koeri.boun.edu.tr/sismo/default.htm) (after Harmandar et al., 2012)

Eq No	Earthquake	Date	Latitude N	Longitude E	GMT	M_L	M_d	Depth (km)	Fault mechanism	Number of recording stations	Maximum Epicentral Distance (km)	Minimum Epicentral Distance (km)
1	Güzelyalı	19/09/2003	40.8498	29.2867	00:51	3.1	3.2	10.3	Strike-slip	16	16	1
2	Yalova	16/05/2004	40.6957	29.3222	03:30	4.3	4.2	9.1	Strike-slip	72	58	14
3	Marmara Sea	29/09/2004	40.7797	29.0200	15:42	4.0	-	8.3	Strike-slip	86	34	14
4	Kuşgölü	20/10/2006	40.2635	27.9843	21:15	-	5.2	5.4	Strike-slip	43	130	101
5	Gemlik	24/10/2006	40.4240	28.9947	17:00	-	5.2	9.2	Strike-slip	47	70	52
6	Çınarcık	12/03/2008	40.6210	29.0110	20:52	4.8	-	8.9	Normal	54	50	30

M_d : Earthquake Duration Magnitude, M_L : Local Magnitude

The site classification map for Istanbul is prepared by OYO International Cooperation within the microzonation project of the Istanbul Metropolitan Municipality for the European and Asian parts of Istanbul. The distribution of average shear wave velocity for the top 30 m of soil (V_{S30}) distribution map is presented in Figure 2 and Figure 3. This map shows that most part of the south part of the European side have low V_{S30} values. The Asian region has stiffer soil conditions and has comparatively high shear wave velocities. V_{S30} values for each station is obtained rom the project and used in the regression analysis to obtain the empirical coherency model.

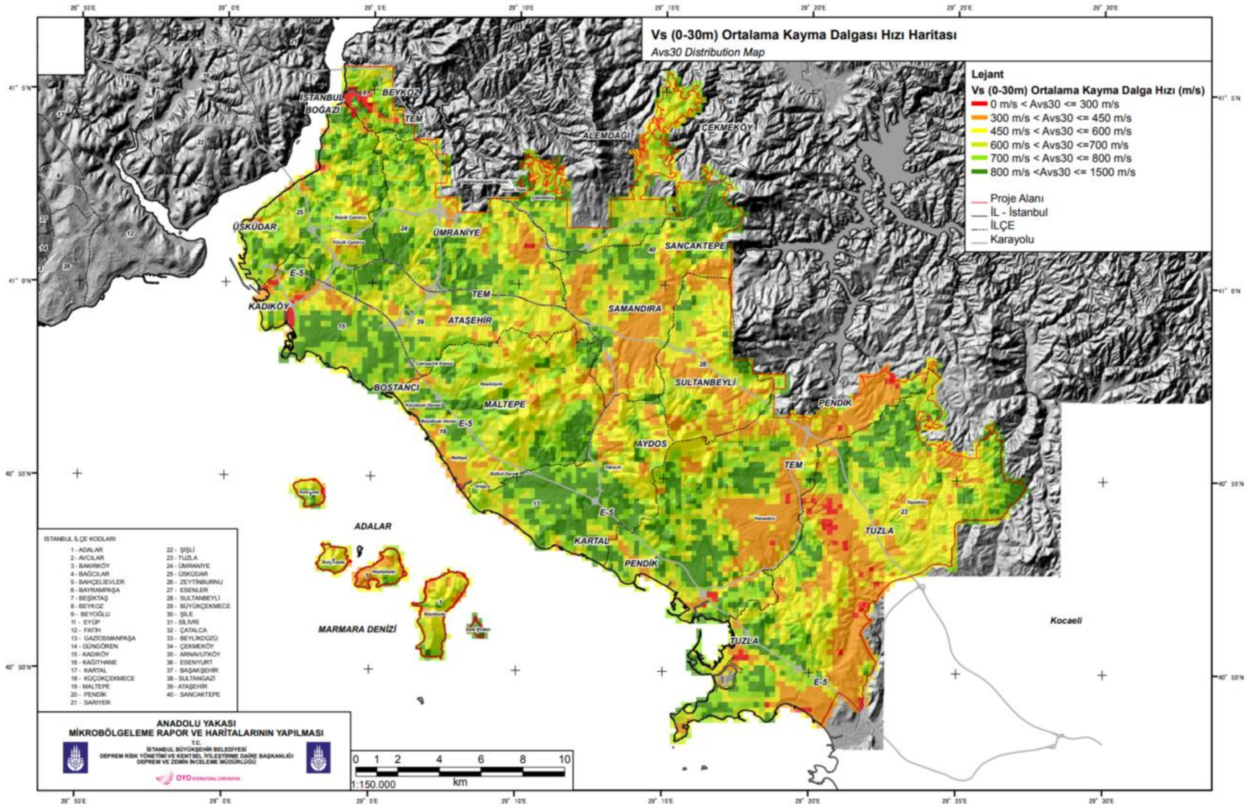

Figure 2. V_{S30} map of Istanbul - Asian Side (Istanbul Metropolitan Municipality-OYO)

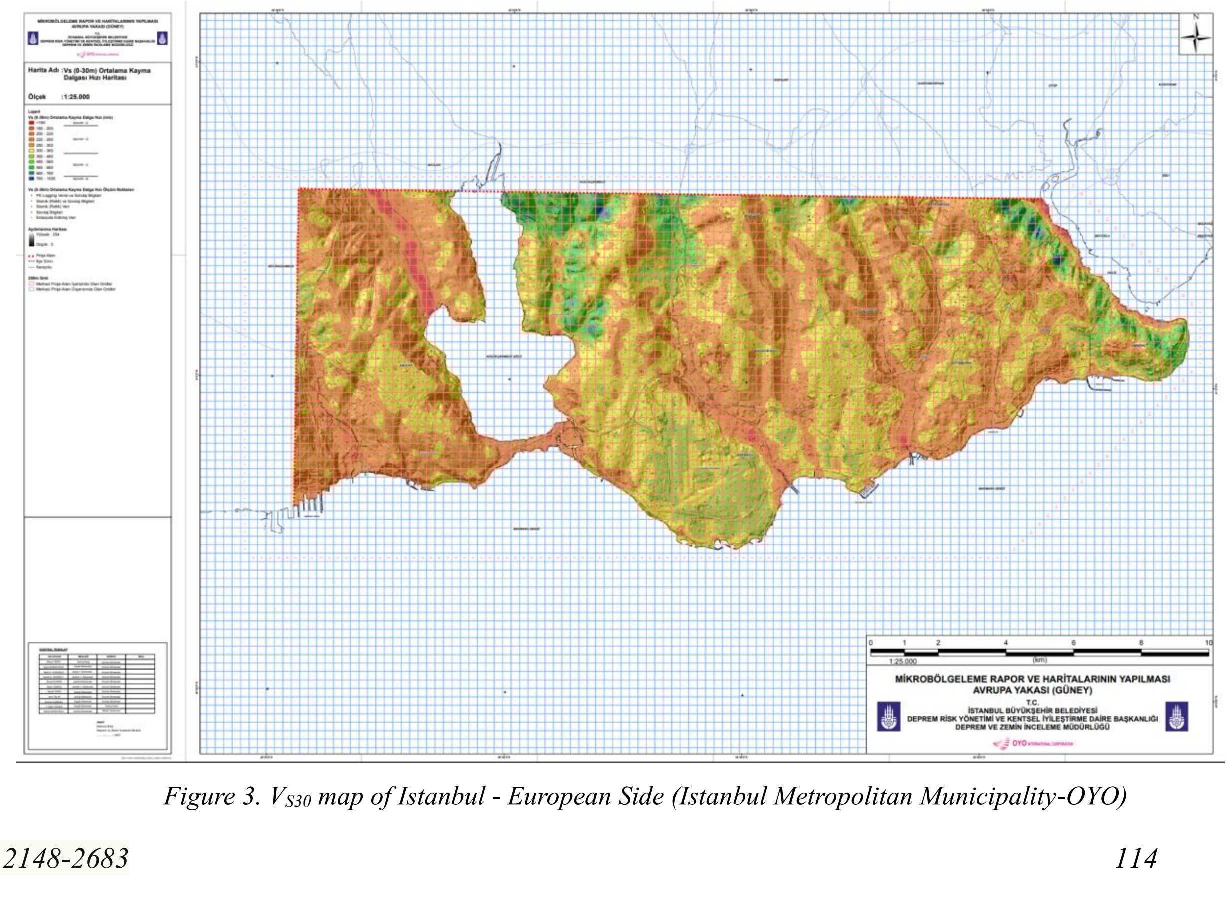

Figure 3. V_{S30} map of Istanbul - European Side (Istanbul Metropolitan Municipality-OYO)

3. Results and Discussion

3.1. Results and Discussion

The coherency values for distance bins (Less than 2.0 km; between 2.0 and 2.5 km; between 2.5 and 3.0 km; between 3.0 and 3.5 km; between 3.5 and 4.0 km; between 4.0 and 4.5 km; and between 4.5 and 5.0 km) associated with September, 19 2003; May 16, 2004; September, 29 2004; October 20, 2006; October 24, 2006; and March 12, 2008 earthquakes are calculated. 332 ground motion data are used, totally. 9837 sets are utilized to obtain the coherency values by Equation 1. For the brevity, only average coherency values of May 16, 2004 for EW component are demonstrated in Figure 4. The variation of coherencies with both distance and frequency in 3-D are represented in Figure 5 for September, 29 2004 earthquake. As expected, coherency values for all events generally decrease with the increase of separation distance and frequency. The coherency values in terms of distance and frequency is clearly presented. The reason is attributed to higher number of earthquake ground motion data in this earthquake. However, in some cases, this may change. Coherency values may increase when the separation distance increase for different frequency ranges. The reason is due to the lack of recorded data at some earthquakes (Table 1) and variation of the station locations from earthquake to earthquake.

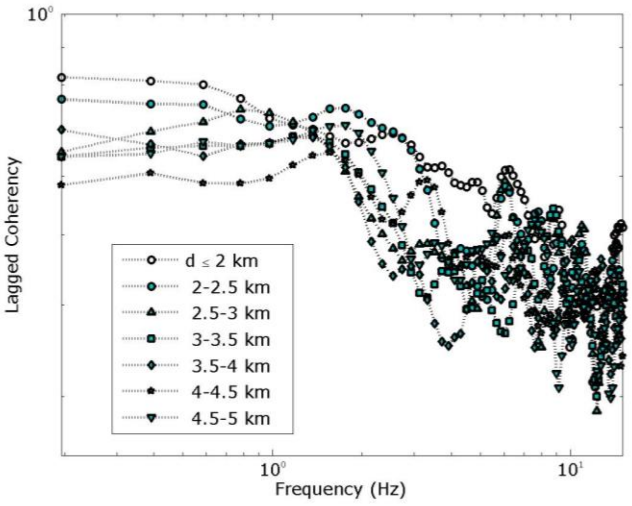

Figure 4. Average coherency values for EW direction (11-point) – May 16, 2004 earthquake

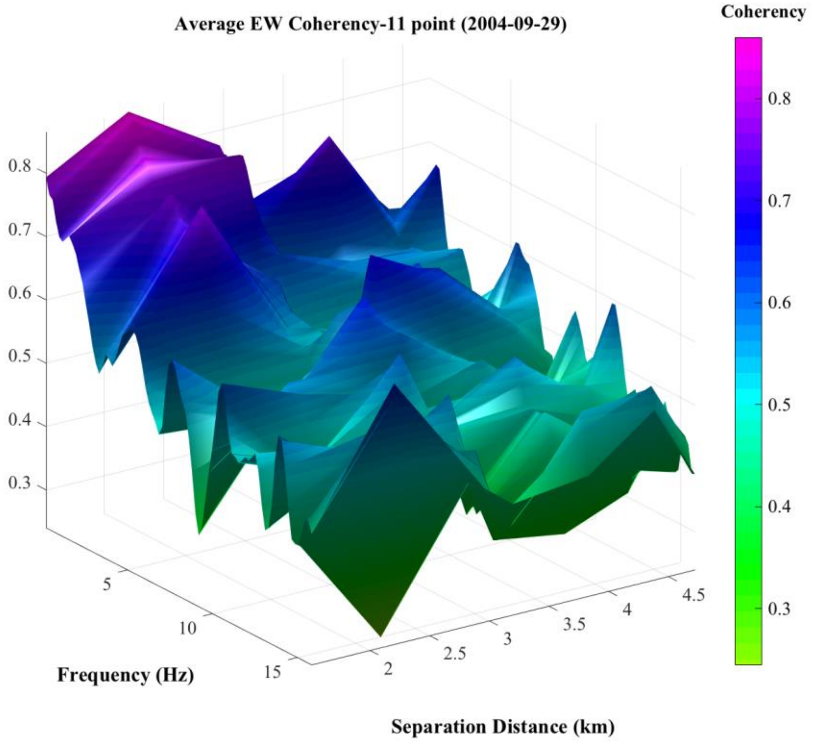

Figure 5. Average coherency values of September, 29 2004 earthquake recorded by IERRS with respect to separation distance and frequency

3.2. Nonlinear Regression Analysis

The decrease of coherency with respect to frequency is approximately exponential. Therefore, the formula is selected in exponential form. In this study, the purpose is to examine the dependency of coherency on V_{S30}, frequency and separation distance between the stations. Several trials of formula for nonlinear regression analysis have been done to select the appropriate and accurate coherency function considering the bias and standard deviation. Eventually, the following lagged coherency function is established for EW and NS components of earthquake ground motion data:

$$\gamma_{ij}(f,d) = e^{-(bfd)^2 V_{S30ij}} + \sigma \qquad (4)$$

where γ_{ij} is the coherency, b is the regression coefficient, d is the station separation distance, f is the frequency, V_{S30ij} is the multiplication of V_{S30} values of i^{th} and j^{th} stations and σ is the standard deviation. Firstly, regression analyses are achieved for six earthquakes, seperately. Then, the analyses are carried out for whole data. The regression coefficient, b, is listed for each earthquake and whole data set considering EW and NS components in Table 2. It is seen that the regression coefficients, b, are close to each other for every earthquake and whole earthquake dataset. Also, the values are nearly same for both EW and NS components.

Table 2. Regression coefficients based on Equation 4 for data recorded by IERRS

Regression Coefficient	2003.09.19 earthquake	2004.05.16 earthquake	2004.09.29 earthquake	2006.10.20 earthquake	2006.10.24 earthquake	2008.03.12 earthquake	All data
b_{EW}	0.0017	0.0017	0.0010	0.0015	0.0019	0.0013	0.0013
b_{NS}	0.0018	0.0018	0.0011	0.0019	0.0020	0.0014	0.0015

Comparison of observed coherency data with coherency model for separation distance less than 1 km is represented in Figure 6. As it is expected, coherency values decrease with the increase of frequency. The derived empirical model has a good fit with observed data with all distance bins. For the brevity, only the values for the separation distance less than 1 km is considered and shown. The difference between the EW and NS components is small. After then, only results from EW components are evaluated.

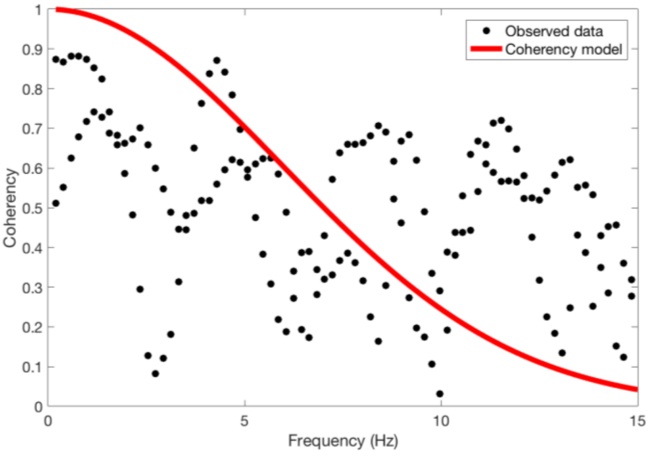

Figure 6. Comparison of coherency model with respect to observed coherency values of EW component for separation distance less than 1 km

3.3. Evaluation of change in separation distance

The derived empirical coherency model is tested for the variation of frequency and separation distance. Figure 7 represents the comparison of coherency in terms of different separation distances (500m, 1000m) and different V_{S30ij} values. When the V_{S30ij} values are stationary, the coherency model decays with the increase in the separation distance (Figure 7). The coherency values at 20 Hz differ dramatically when the separation distance scaled with 5. It is expected that the relation between two data is more coherent if the separation distance is less. Because the wave passage effect, incoherence effect and the difference between the soil profile, so the local site effects will be nearly similar.

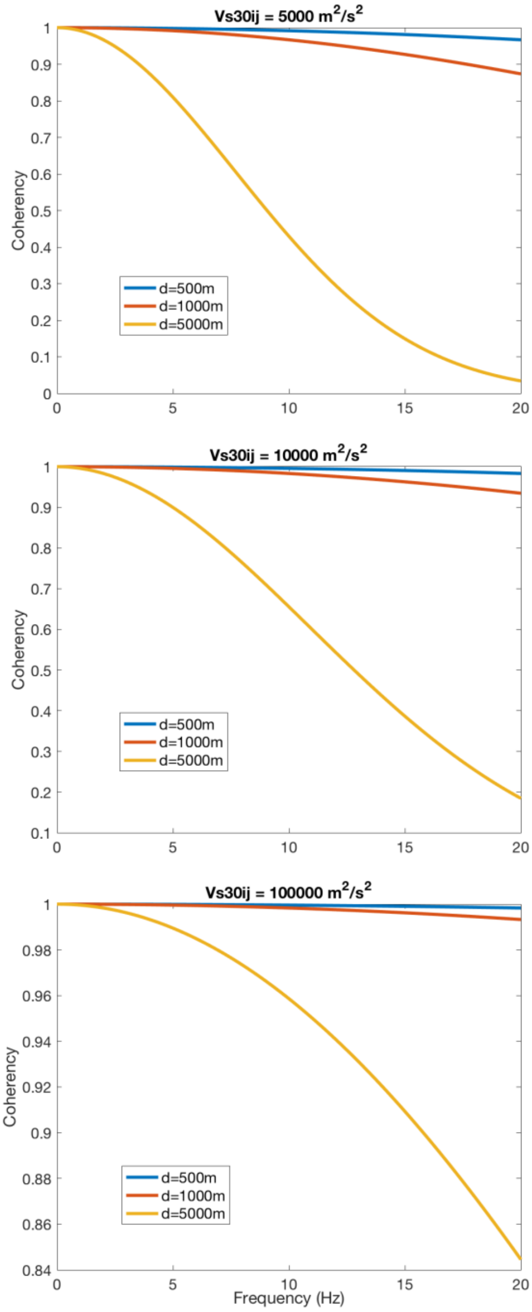

Figure 7. Comparison of coherency model in terms of separation distance considering V_{S30ij} values stationary. V_{S30ij} value is the multiplication of V_{S30} values of i^{th} and j^{th} stations.

3.4. Evaluation of change in V_{S30}

The derived coherency model is tested for the parametric analysis. To understand the change of coherency values in terms of V_{S30ij} values, Figure 8 is drawn in three parts considering the separation distance stationary for each, but this time V_{S30ij} values vary. Figure 8 represents the comparison of coherency in terms of different V_{S30ij} values (5000 m^2/s^2, 10000 m^2/s^2, 100000 m^2/s^2). Only coherency model for EW components is demonstrated in Figure 8. Besides, the difference between the coherency model generated using data from EW and data from NS components is very small.

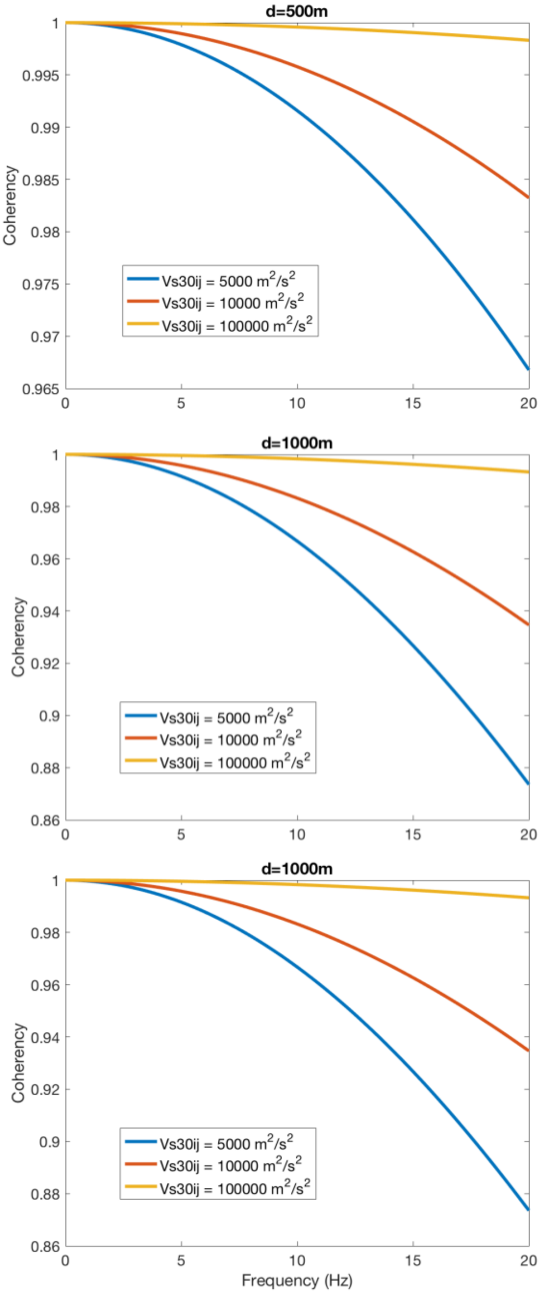

Figure 8. Comparison of coherency model in terms of V_{S30ij} values considering separation distance stationary. V_{S30ij} value is the multiplication of V_{S30} values of i^{th} and j^{th} stations.

Coherency values doesn't change dramatically when separation distance changes from d =500m to d=1000m. However, when the separation distance taken as 5000m, coherency values decreases related withV_{S30}, clearly. The coefficient between V_{S30} values of i^{th} and j^{th} stations increases with increase of coherency values. The variation between V_{S30ij} values at low frequencies is small. It expands at high frequencies. In other words, coherency data is affected by V_{S30ij} at short periods much more than the long periods.

3.5. Comparison of model with Luco & Wong (1986)'s model

The proposed model is compared with Luco and Wong (1986) to investigate the relation between the model and literature. Luco and Wong (1986) proposed a model which is based on the analysis of shear waves considering the propagation in random medium. It has an exponential decay in terms of separation distance between stations and frequency. Additionally, Luco and Wong (1986)'s model is used and referenced mostly in literature. Also, their formula is based on shear waves which are related to the site properties of the region.

In the comparative analysis, same separation distance and V_{S30ij} parameter is selected. Figure 9 represents the comparison of these two models. The trend between both models is close to each other, generally. In details, proposed model has higher values at frequency approximately 6 Hz than Luco and Wong (1986)'s model. The reason for this is soil parameter used in the generation of model which changes. The derived model in this study uses V_{S30ij}. On the other hand, Luco and Wong's model is related with V_S. By the decrease and increase of frequency, the variation between those gets small.

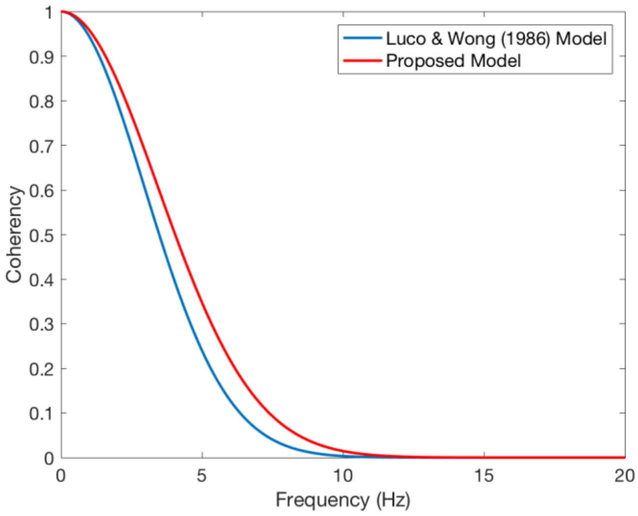

Figure 9. Comparison of proposed coherency model with the model by Luco and Wong (1986)

4. Conclusions

In this study, a coherency model based on V_{S30} is proposed as a function of frequency and separation distance for Istanbul. Data from Istanbul Earthquake Rapid Response System are used in the nonlinear regression analysis. Six earthquakes triggered by more stations are selected to derive the model. Nonlinear regression analysis is achieved for data from EW and NS components. Regression coefficient, b, is derived for each earthquake and component. Regression coefficient, b, for every earthquake differs, but not dramatically. This change is caused by the recorded earthquake data at each earthquake. Number of stations trigerred during an earthquake is much more than the other earthquake. This affects the separation distance. The model is valid for the magnitude range of 3 to 5 and the separation distance range of 0.5 km to 5 km.

As expected, the coherency model decreases with increase of separation distance and frequency. The influence of parameter V_{S30ij} is significant on coherency values. When the parameter V_{S30ij} decays, coherency values also decrease by separation distance and frequency. A significant change does not observed in the coherency model for one earthquake to another. Additionally, coherency model derived by using data from EW components does not have an extreme difference from the model created by data from NS component. The comparative study done with the literature shows that the proposed model has a good compatibility with the published model. The variation is high for medium frequencies. Meanwhile, the similarity increases for short and long periods. These observations are valid for both EW and NS components.

The proposed coherency model can be use for the regions exposed to moderate earthquakes. Considering a reference earthquake ground motion data, less coherent seismic data will obtain for soft soils; on the contrary, more coherent data for hard soils depending on V_{S30} values. Additionally, it may use to generate earthquake ground motion data compatible with design spectrum.

The main purpose of the derivation of coherency model here is to lead produce spatially variable ground motions for the design of earthquake resistant structures. The derived model based on V_{S30} can be utilized for the generation of non-uniform earthquake ground motion data. The comparative analysis shows that this model can be used at any region for the valid separation distance and frequency with specified V_{S30} values.

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